Math 1A (9:30am – 10:20am)
Quiz 5
Thu Feb 11, 2010

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

The time (t) required for a disc of liquid metal to solidify depends on the thickness (d) of the disc. If $t = f(d)$, where t is measured in minutes, and d is measured in millimeters, give the practical meaning, including units, for the statement $f'(2) = 6$. **SCORE: ___ / 2 POINTS**

IF A DISC OF LIQUID METAL IS 2mm THICK,
IT WILL TAKE 6 MINUTES LONGER FOR IT TO SOLIDIFY
FOR EACH ADDITIONAL mm IN ITS THICKNESS

State the Intermediate Value Theorem. **NO PARTIAL CREDIT.** **SCORE: ___ / 1 POINT**

SEE YOUR NOTES / TEXTBOOK

Using the definition of the derivative, find the derivative of $f(x) = \cos x$. **SCORE: ___ / 3 POINTS**
You may use the values of the 2 trigonometric limits discussed in class yesterday without proving them.
DO NOT USE THE DERIVATIVES OF ANY TRIGONOMETRIC FUNCTIONS.

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} \quad \frac{1}{2}$$

$\frac{1}{2}$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \quad 1$$

$$= \lim_{h \to 0} \left(\cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \left(\frac{\sin h}{h} \right) \right) \quad 1$$

$$= \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

CONTINUED ON OTHER SIDE ➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜➜

Find the requested derivatives using any of the differentiation shortcuts discussed so far. **SCORE: ___ / 14 POINTS**

Simplify your answers. Factor where appropriate.

[a] If $f(x) = \cot x^5$, find $f''(x)$. **BE CAREFUL.**

$$f'(x) = (-\csc^2 x^5)(5x^4) = \boxed{-\boxed{5x^4}\csc^2 x^5} \quad \tfrac{1}{2} \quad \tfrac{1}{2}$$

$$f''(x) = -20x^3\csc^2 x^5 - 5x^4 \boxed{(2\csc x^5)}\boxed{(-\csc x^5 \cot x^5)}\boxed{(5x^4)} \quad \tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2}$$

$$= \tfrac{1}{2}\ \boxed{\boxed{-20x^3\csc^2 x^5} + 50x^8\csc^2 x^5 \cot x^5} \quad \tfrac{1}{2}$$

$$= \boxed{-10x^3\csc^2 x^5(2 - 5x^5\cot x^5)} \quad \tfrac{1}{2}$$

[b] If $m(x) = \dfrac{1}{\sqrt{2^x + \sec x}}$, find $m'(0)$.

$$m'(x) = \boxed{-\tfrac{1}{2}(2^x + \sec x)^{-\frac{3}{2}}}\left(\boxed{(\ln 2)2^x} + \boxed{\sec x \tan x}\right) \quad 1 \quad 1 \quad \tfrac{1}{2}$$

$$m'(0) = -\tfrac{1}{2}(1+1)^{-\frac{3}{2}}(\ln 2 + 1 \cdot 0)$$

$$= -\frac{\ln 2}{2 \cdot 2^{\frac{3}{2}}}$$

$$= \boxed{-\frac{\ln 2}{4\sqrt{2}} = -\frac{\sqrt{2}\ln 2}{8}} \quad \tfrac{1}{2}$$

EITHER ONE

[c] If $g(x) = \dfrac{\tan x + \cot x}{\csc x}$, find $g'(x)$.

$$g'(x) = \boxed{\frac{(\sec^2 x - \csc^2 x)(\cancel{\csc} x) - (\tan x + \cot x)(-\cancel{\csc} x \cot x)}{\csc^{\cancel{2}} x}} \quad 2$$

SEE OTHER KEY FOR ALTERNATE SOLUTION

$$= \frac{\sec^2 x - \csc^2 x + 1 + \cot^2 x}{\csc x}$$

$$= \frac{\sec^2 x - \cancel{\csc^2 x} + \cancel{\csc^2 x}}{\csc x} = \boxed{\frac{\sec^2 x}{\csc x}} \quad \tfrac{1}{2}$$

$$= \frac{1}{\cos^2 x} \cdot \sin x = \frac{\sin x}{\cos^2 x} = \boxed{\sec x \tan x} \quad \tfrac{1}{2}$$

[d] If $f(t) = e^{at}\sin bt$, find $f''(t)$.

$$f'(t) = e^{at}(a)\sin bt + e^{at}\cos bt(b)$$

$$\overset{1}{=} \boxed{ae^{at}\sin bt} + \boxed{be^{at}\cos bt} \quad 1$$

$$f''(t) = ae^{at}(a)\sin bt + ae^{at}\cos bt(b) + be^{at}(a)\cos bt + be^{at}(-\sin bt)b$$

$$\overset{1}{=} \boxed{(a^2 - b^2)e^{at}\sin bt} + \boxed{2abe^{at}\cos bt} \quad 1$$