Math 1A (9:30am – 10:20am)
Quiz 5
Thu Feb 11, 2010

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

The value (v) of a bottle of wine depends on its Wine Spectator rating (r). If $v = f(r)$, where v is measured in dollars, and r is measured in points, give the practical meaning, including units, for the statement $f'(87) = 4$. **SCORE: ___ / 2 POINTS**

IF A BOTTLE OF WINE HAS AN 87 POINT RATING, ITS VALUE INCREASES \$4 FOR EACH ADDITIONAL POINT IN ITS RATING.

State the Intermediate Value Theorem. **NO PARTIAL CREDIT.** **SCORE: ___ / 1 POINT**

SEE YOUR TEXTBOOK / NOTES

Using the definition of the derivative, find the derivative of $f(x) = \cos x$. **SCORE: ___ / 3 POINTS**
You may use the values of the 2 trigonometric limits discussed in class yesterday without proving them.
DO NOT USE THE DERIVATIVES OF ANY TRIGONOMETRIC FUNCTIONS.

$$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} \quad \tfrac{1}{2}$$

$\tfrac{1}{2}$

$$= \lim_{h\to 0} \frac{\cos x \cosh - \sin x \sin h - \cos x}{h} \quad 1$$

$$= \lim_{h\to 0} \left(\cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \cdot \frac{\sin h}{h} \right) \quad 1$$

$$= \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

CONTINUED ON OTHER SIDE ➡

Find the requested derivatives using any of the differentiation shortcuts discussed so far. SCORE: ___ / 14 POINTS
Simplify your answers. Factor where appropriate.

[a] If $f(t) = e^{bt}\cos at$, find $f''(t)$.

$$f'(t) = (e^{bt})(b)(\cos at) + (e^{bt})(-\sin at)(a)$$

$$= \boxed{be^{bt}\cos at}\ \boxed{-ae^{bt}\sin at} \quad 1 \quad 1$$

$$f''(t) = b(e^{bt})(b)(\cos at) + (be^{bt})(-\sin at)(a)$$

$$\quad - a(e^{bt})(b)(\sin at) - (ae^{bt})(\cos at)(a)$$

$$= \boxed{(b^2 - a^2)e^{bt}\cos at}\ \boxed{-2abe^{bt}\sin at} \quad 1 \quad 1$$

[b] If $g(x) = \dfrac{\cot x + \tan x}{\csc x}$, find $g'(x)$.

$$g(x) = \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{1}{\sin x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x}$$

$$= \boxed{\sec x} \quad 2$$

$$g'(x) = \boxed{\sec x \tan x} \quad 1$$

SEE OTHER KEY FOR ALTERNATE SOLUTION

[c] If $m(x) = \dfrac{1}{\sqrt{3^x + \sec x}}$, find $m'(0)$.

$$m'(x) = \underbrace{\boxed{-\tfrac{1}{2}(3^x + \sec x)^{-\frac{3}{2}}}}_{1}\left(\underbrace{\boxed{(\ln 3)3^x}}_{1} + \underbrace{\boxed{\sec x \tan x}}_{\frac{1}{2}}\right)$$

$$m'(0) = -\tfrac{1}{2}(1+1)^{-\frac{3}{2}}(\ln 3 + 1 \cdot 0)$$

$$= -\frac{\ln 3}{2 \cdot 2^{\frac{3}{2}}}$$

$$= \boxed{-\frac{\ln 3}{4\sqrt{2}} = -\frac{\sqrt{2}\ln 3}{8}} \quad \tfrac{1}{2}$$

EITHER ONE

[d] If $f(x) = \tan x^4$, find $f''(x)$. **BE CAREFUL.**

$$f'(x) = (\sec^2 x^4)(4x^3) = \underbrace{\boxed{4x^3}}_{\frac{1}{2}}\underbrace{\boxed{\sec^2 x^4}}_{\frac{1}{2}}$$

$$f''(x) = 12x^2\sec^2 x^4 + (4x^3)\left(\underbrace{\boxed{2\sec x^4}}_{\frac{1}{2}}\right)\left(\underbrace{\boxed{\sec x^4 \tan x^4}}_{\frac{1}{2}}\right)\underbrace{\boxed{4x^3}}_{\frac{1}{2}}$$

$$= \underbrace{\boxed{12x^2\sec^2 x^4}}_{\frac{1}{2}}\ \underbrace{\boxed{+ 32x^6\sec^2 x^4 \tan x^4}}_{\frac{1}{2}}$$

$$= \underbrace{\boxed{4x^2\sec^2 x^4(3 + 8x^4\tan x^4)}}_{\frac{1}{2}}$$