Math 1A (9:30am – 10:20am)
Quiz 6
Fri Feb 19, 2010

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

The Rotten Tomatoes rating (r) of a movie depends on the amount of money (A) spent on the critics' dinner. **SCORE: ___ / 2 POINTS**
If $r = f(A)$, where r is measured in points, and A is measured in thousands of dollars, give the practical meaning, including units, for the statement $f'(12) = 0.5$.

IF \$12,000 IS SPENT ON THE CRITICS' DINNER FOR A MOVIE, THE ROTTEN TOMATOES RATING WOULD RISE BY $\frac{1}{2}$ POINT FOR EACH ADDITIONAL \$1,000 SPENT ON THE DINNER.

State the Intermediate Value Theorem. **NO PARTIAL CREDIT.** **SCORE: ___ / 1 POINT**

SEE YOUR TEXT / NOTES

If $x^3 - y^3 = 7$ and $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = 2$. **SCORE: ___ / 3 POINTS**

$3x^2\frac{dx}{dt} - 3y^2\frac{dy}{dt} = 0$ 2

$3 \cdot 2^2 \cdot 3 - 3 \cdot 1^2 \frac{dy}{dt} = 0$

$\frac{dy}{dt} = 12$ $\frac{1}{2}$

$2^3 - y^3 = 7$

$-y^3 = -1$

$y = 1$ WHEN $x = 2$ $\frac{1}{2}$

CONTINUED ON OTHER SIDE ➡

Find the requested derivatives using any of the differentiation shortcuts discussed so far. **SCORE: ___ / 14 POINTS**

Simplify your answers. Factor where appropriate.

[a] If $f(x)=\sqrt{1-9x^2}\sin^{-1}3x$, find $f'(x)$.

$f'(x)=\frac{1}{2}(1-9x^2)^{-\frac{1}{2}}(-18x)\sin^{-1}3x+\sqrt{1-9x^2}\,\frac{1}{\sqrt{1-(3x)^2}}\cdot 3$ (1, 1)

$=\frac{-9x\sin^{-1}3x}{\sqrt{1-9x^2}}+3$ (1)

[b] If $\cos xy=x+\sin y$, find $\frac{dy}{dx}$.

$(-\sin xy)\left(y+x\frac{dy}{dx}\right)=1+\cos y\frac{dy}{dx}$ ($\frac{1}{2}$, 1, 1)

$-y\sin xy-x\sin xy\frac{dy}{dx}=1+\cos y\frac{dy}{dx}$

$-y\sin xy-1=(x\sin xy+\cos y)\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{-y\sin xy-1}{x\sin xy+\cos y}$ ($1\frac{1}{2}$)

[c] If $f(x)=\ln[(1-2x^2)^6 e^{\arctan x}]$, find $f'(x)$.

$f'(x)=\frac{1}{(1-2x^2)^6 e^{\arctan x}}\cdot\left[6(1-2x^2)^5(-4x)e^{\arctan x}+(1-2x^2)^6\frac{e^{\arctan x}}{1+x^2}\right]$ ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$)

$=\frac{1}{1-2x^2}\left(-24x+\frac{1-2x^2}{1+x^2}\right)$

$=\frac{-24x}{1-2x^2}+\frac{1}{1+x^2}$ ($1\frac{1}{2}$)

OR

$f(x)=\ln(1-2x^2)^6+\ln e^{\arctan x}$

$=6\ln(1-2x^2)+\arctan x$ (1)

$f'(x)=6\cdot\frac{1}{1-2x^2}(-4x)+\frac{1}{1+x^2}$ (1, $\frac{1}{2}$)

$=\frac{-24x}{1-2x^2}+\frac{1}{1+x^2}$ ($\frac{1}{2}$)

[d] If $y=(\sin x)^x$, find $\frac{dy}{dx}$.

$\ln y=x\ln\sin x$ (1)

$\frac{1}{y}\frac{dy}{dx}=\ln\sin x+x\cdot\frac{1}{\sin x}\cdot\cos x$ ($\frac{1}{2}$)

$=\ln\sin x+x\cot x$ (1)

$\frac{dy}{dx}=y(\ln\sin x+x\cot x)$ ($\frac{1}{2}$)

$=(\sin x)^x(\ln\sin x+x\cot x)$ (1)