Math 1A (9:30am – 10:20am)
Quiz 6
Fri Feb 19, 2010

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

The number of tickets (t) sold on the opening day of a movie depends on the amount of money (A) spent on advertising. If $t = f(A)$, where t is measured in thousands of tickets, and A is measured in millions of dollars, give the practical meaning, including units, for the statement $f'(10) = 110$.

SCORE: ___ / 2 POINTS

IF \$10,000,000 IS SPENT TO ADVERTISE A MOVIE, 110,000 EXTRA TICKETS WILL BE SOLD FOR EACH ADDITIONAL \$1,000,000 SPENT ON ADVERTISING

State the Intermediate Value Theorem. **NO PARTIAL CREDIT.**

SCORE: ___ / 1 POINT

SEE YOUR TEXT / NOTES

If $x^3 + y^3 = 9$ and $\frac{dx}{dt} = 2$, find $\frac{dy}{dt}$ when $x = 1$.

SCORE: ___ / 3 POINTS

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$ 2

$3 \cdot 1^2 \cdot 2 + 3 \cdot 2^2 \frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{1}{2}$ $\frac{1}{2}$

$1^3 + y^3 = 9$
$y^3 = 8$
$y = 2$ WHEN $x = 1$
$\frac{1}{2}$

CONTINUED ON OTHER SIDE ➡

Find the requested derivatives using any of the differentiation shortcuts discussed so far. **SCORE: ___ / 14 POINTS**

Simplify your answers. Factor where appropriate.

[a] If $y = (\sin x)^x$, find $\frac{dy}{dx}$.

$\ln y = x \ln \sin x$ 1

$\frac{1}{y}\frac{dy}{dx} = \ln \sin x + x \cdot \frac{1}{\sin x}\cos x$ $\frac{1}{2}$

$= \ln \sin x + x \cot x$ 1

$\frac{dy}{dx} = y(\ln \sin x + x \cot x)$ $\frac{1}{2}$

$= (\sin x)^x (\ln \sin x + x \cot x)$ 1

[b] If $f(x) = \ln[(1-3x^2)^4 e^{\arctan x}]$, find $f'(x)$.

$f(x) = \ln(1-3x^2)^4 + \ln e^{\arctan x}$

$= 4\ln(1-3x^2) + \arctan x$ 1 OR

$f'(x) = 4 \cdot \frac{1}{1-3x^2} \cdot -6x + \frac{1}{1+x^2}$ 1 $\frac{1}{2}$

$= \frac{-24x}{1-3x^2} + \frac{1}{1+x^2}$ $\frac{1}{2}$

↳ $f'(x) = \frac{1}{(1-3x^2)^4 e^{\arctan x}} \cdot [4(1-3x^2)^3(-6x)e^{\arctan x} + (1-3x^2)^4 e^{\arctan x}\frac{1}{1+x^2}]$ $\frac{1}{2}$ $\frac{1}{2}$

$= \frac{1}{1-3x^2}(-24x + \frac{(-3x^2)}{1+x^2})$ $\frac{1}{2}$

$= \frac{-24x}{1-3x^2} + \frac{1}{1+x^2}$ $\frac{1}{2}$

[c] If $f(x) = \sqrt{1-9x^2}\sin^{-1} 3x$, find $f'(x)$.

$f'(x) = \frac{1}{2}(1-9x^2)^{-\frac{1}{2}}(-18x)\sin^{-1} 3x + \sqrt{1-9x^2}\frac{1}{\sqrt{1-(3x)^2}} \cdot 3$ 1 1

$= \frac{-9x\sin^{-1} 3x}{\sqrt{1-9x^2}} + 3$ 1

[d] If $\sin xy = x + \cos y$, find $\frac{dy}{dx}$.

$(\cos xy)(y + x\frac{dy}{dx}) = 1 - (\sin y)\frac{dy}{dx}$ $\frac{1}{2}$ 1 1

$y\cos xy + x\cos xy\frac{dy}{dx} = 1 - \sin y\frac{dy}{dx}$

$(x\cos xy + \sin y)\frac{dy}{dx} = 1 - y\cos xy$

$\frac{dy}{dx} = \frac{1 - y\cos xy}{x\cos xy + \sin y}$ $1\frac{1}{2}$