

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your social security number ?

[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER,
USE YOUR STUDENT ID NUMBER]**NO CALCULATORS ALLOWED**Find the global maximum and minimum of $f(x) = 2x - 3x^{\frac{2}{3}}$ on $[-1, 3]$.

SCORE: ___ / 4 POINTS

You may use the following approximations in your work.

x	2	3	4	5	6	7	8	9	10
$\sqrt[3]{x}$	1.3	1.4	1.6	1.7	1.8	1.9	2	2.1	2.2

$$f'(x) = \frac{1}{2} \left[2 - 2x^{-\frac{1}{3}} \right] \text{ IS UNDEFINED @ } \boxed{x=0} \in \text{DOMAIN OF } f$$

$$\frac{1}{2} \quad 2 - 2x^{-\frac{1}{3}} = 0$$

$$x^{-\frac{1}{3}} = 1$$

$$\boxed{x=1}$$

x	$f(x)$
$\frac{1}{4} \quad \boxed{-1}$	$-2 - 3 = -5$
$\frac{1}{4} \quad \boxed{0}$	0
$\frac{1}{4} \quad \boxed{1}$	$2 - 3 = -1$
$\frac{1}{4} \quad \boxed{3}$	$6 - 3\sqrt[3]{9} \approx -0.3$

GLOBAL MAX @ $x=0$, $\frac{1}{2}$
 MIN @ $x=-1$, $\frac{1}{2}$

$\frac{1}{4}$ FOR EACH x OTHER THAN
 CRITICAL #'S AND ENDPOINTS

For the function $f(x) = \frac{1}{x-4}$ on the interval $[0, 3]$, find all values of c which satisfy the conclusion of the Mean Value Theorem. SCORE: ___ / 3 POINTS

$$f'(x) = -\frac{1}{(x-4)^2}$$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{-1 - \frac{1}{4}}{3} = \frac{-\frac{5}{4}}{3} = -\frac{5}{12}$$

$$\boxed{-\frac{1}{(c-4)^2} = -\frac{5}{12}}$$

$$(c-4)^2 = 4$$

$$c-4 = \pm 2$$

$$c = 4 \pm 2 = 6 \text{ OR } 2$$

$-\frac{1}{2}$ IF YOU ALSO
 SAID $c=6$

$$\rightarrow \boxed{c=2} \in (0, 3)$$

FOR ALL REMAINING QUESTIONS, YOU WILL BE GIVEN INFORMATION
 ABOUT f' OR f'' BUT YOU MUST ANSWER QUESTIONS ABOUT f .

The second derivative of a polynomial f is $f''(x) = 3x^2 - 6x$. The critical numbers of f are -1 and 2 . SCORE: ___ / 4 POINTS

- [a] What does the Second Derivative Test tell you about each critical number above?

$$f''(-1) = 3 + 6 = 9 > 0 \text{ LOCAL MIN } \frac{1}{2}$$

$$f''(2) = 12 - 12 = 0 \text{ NO CONCLUSION } \frac{1}{2}$$

- [b] Find all intervals over which f is concave up.

$$f''(x) = 3x(x-2) = 0 @ x = 0, 2 \quad \frac{1}{2}$$

f''	+	-	+	$\frac{1}{2}$
$3x$	-	+	+	
$x-2$	-	-	+	

f IS CONCAVE UP ON $(-\infty, 0) \quad \frac{1}{2}$
AND $(2, \infty) \quad \frac{1}{2}$

The derivative of a function f is $f'(x) = \frac{(x+2)^4}{\sqrt[3]{x}}$. f is continuous on $(-\infty, \infty)$.

SCORE: ___ / 3 POINTS

- [a] Find the critical numbers of f .

$$f' = 0 @ x = -2 \quad \frac{3}{4}$$

$$f' \text{ UNDEFINED @ } x = 0 \quad \frac{3}{4}$$

- [b] Using the First Derivative Test, determine if each critical number is a local maximum or local minimum or neither.

f'	-	-	+	$\frac{1}{2}$
$(x+2)^4$	+	+	+	
$\sqrt[3]{x}$	-	-	+	

$x = 0$ IS LOCAL MIN. $\frac{1}{2}$

$x = -2$ IS NEITHER. $\frac{1}{2}$

The graph of the derivative f' of a continuous function f is given on the right.

SCORE: ___ / 6 POINTS

- [1] Find the critical numbers of f . Give a brief explanation of your answer.

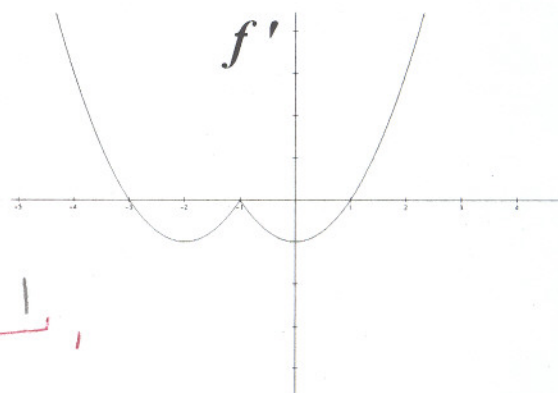
$$f' = 0 @ x = -3, -1, 1$$

$$f' \text{ IS NEVER UNDEFINED}$$

- [2] Find the x-coordinates of all local minima of f .

Give a brief explanation of your answer.

$$f' \text{ CHANGES FROM - TO + @ } x = 1$$



- [3] Find all intervals over which f is concave down.

Give a brief explanation of your answer.

$$f' \text{ IS DECREASING ON } (-\infty, -2) \text{ AND } (-1, 0)$$