Math 1A (9:30am – 10:20am)
Quiz 8
Fri Mar 12, 2010

SCORE: ___ / 20 POINTS

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your social security number ? ___ ___
[IF YOU DO NOT HAVE A SOCIAL SECURITY NUMBER, USE YOUR STUDENT ID NUMBER]

NO CALCULATORS ALLOWED

Graph $f(x)=\dfrac{x-2}{x^3}$ using the full procedure discussed in class. Show all relevant work. **SCORE: ___ / 8 POINTS**

Identify the domain, discontinuities (including type), intercepts, asymptotes, local extrema, inflection points, vertical and horizontal tangent lines, intervals of increase/decrease, intervals of upward/downward concavity.

DOMAIN $\{x \neq 0\}$

DISCONTINUITY @ $x=0$

$\lim\limits_{x\to 0^+} \frac{x-2}{x^3} = -\infty$ $\left(\frac{-2}{0^+}\right)$ $\lim\limits_{x\to 0^-} \frac{x-2}{x^3} = \infty$ $\left(\frac{-2}{0^-}\right)$ V.A. @ $x=0$

$f(0)$ DNE — NO y-INT

$\frac{x-2}{x^3}=0$ IF $x=2$ $(2,0)$ x-INT

$\lim\limits_{x\to\infty} \frac{x-2}{x^3} = \lim\limits_{x\to\infty} \frac{1}{3x^2} = 0$ $\left(\frac{\infty}{\infty}, \frac{1}{\infty}\right)$ $\lim\limits_{x\to-\infty} \frac{x-2}{x^3} = \lim\limits_{x\to-\infty} \frac{1}{3x^2} = 0$ $\left(\frac{-\infty}{-\infty}, \frac{1}{\infty}\right)$

H.A. @ $y=0$

$f(x) = x^{-2} - 2x^{-3}$

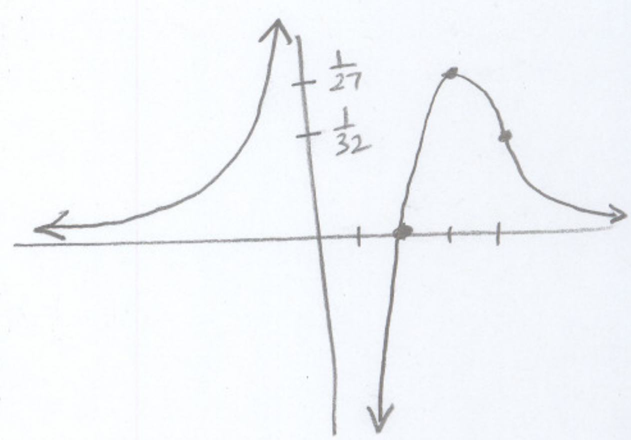
$f'(x) = -2x^{-3} + 6x^{-4} = -2x^{-4}(x-3)$ DNE @ $x=0 \notin$ DOMAIN

$f''(x) = 6x^{-4} - 24x^{-5} = 6x^{-5}(x-4)$ DNE @ $x=0 \notin$ DOMAIN

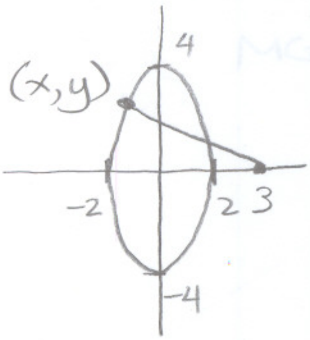
$f'(x)=0$ @ $x=3$ $(3, \frac{1}{27})$

$f''(x)=0$ @ $x=4$ $(4, \frac{1}{32})$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,4)$	4	$(4,\infty)$
f'	+		+	LOCAL MAX	−		−
f''	+		−		−	IP	+
$-2x^{-4}$	−		−		−		−
$x-3$	−		−		+		+
$6x^{-5}$	−		+		+		+
$x-4$	−		−		−		+



Find the points on the ellipse $4x^2 + y^2 = 16$ which are farthest from the point $(3, 0)$. **SCORE: ___ / 6 POINTS**



MINIMIZE d = DISTANCE FROM (x,y) TO $(3,0)$

$$d = \sqrt{(x-3)^2 + y^2}$$

$$d = \sqrt{(x-3)^2 + 16 - 4x^2} = \sqrt{25 - 6x - 3x^2}$$

$$d' = \frac{1}{2\sqrt{25-6x-3x^2}} \cdot (-6-6x)$$

$x \in [-2, 2]$

IS NEVER UNDEFINED ON $[-2,2]$ SINCE $\sqrt{25-6x-3x^2}$ IS THE DISTANCE FROM (x,y) TO $(3,0)$, SO IS NEVER 0 ON $[-2,2]$

x	d
-2	5
-1	$\sqrt{28}$ ← MAX
2	1

$d' = 0$ IF $-6-6x = 0$ IE. $x = -1$

$(-1, \pm\sqrt{12}) = (-1, \pm 2\sqrt{3})$

SEE SHORTER SOLUTION BELOW (BASED ON TEXTBOOK SHORTCUT)

A cylindrical can without a top is made to contain 8π cm^3 of liquid. Find the dimensions that will minimize the metal needed to make the can. **SCORE: ___ / 6 POINTS**

MINIMIZE A = AREA OF SIDE + BOTTOM

$$A = \pi r^2 + 2\pi r h$$

$$V = \pi r^2 h = 8\pi$$

$$h = \frac{8}{r^2}$$

$$A = \pi r^2 + 16\pi r^{-1} \qquad r \in (0, \infty)$$

$$A' = 2\pi r - 16\pi r^{-2}$$ UNDEFINED @ $r = 0$ $\notin$ DOMAIN

$$= 2\pi r^{-2}(r^3 - 8) = 0 \text{ @ } r = 2$$

$$A'' = 2\pi + 32\pi r^{-3} > 0 \text{ ON } (0, \infty)$$ CONCAVE UP

SO $r = 2$cm IS GLOBAL MIN

$h = 2$cm

SHORTER SOLUTION

MINIMIZE $D = d^2$ = SQUARE OF DISTANCE FROM (x,y) TO $(3,0)$

$$D = (x-3)^2 + y^2$$
$$D = (x-3)^2 + 16 - 4x^2$$
$$D = 25 - 6x - 3x^2$$
$$D' = -6 - 6x = 0 \text{ @ } x = -1$$

x	D
-2	25
-1	28 ← MAX
2	1

$$(-1, \pm\sqrt{12}) = (-1, \pm 2\sqrt{3})$$