

Many students lose points for the formal definition of a limit (and other definitions and theorems) because they only focus on the algebra, and ignore the English which connects the algebra. To give you an example of how important the English is, consider this question:

John's car is a Honda Civic. Honda is the manufacturer of Civics and Accords.

Which of the following statements are true ? HINT: Exactly four (4) statements are true.

- [a] John's car is a Honda and John's car is a Civic.
- [b] If John's car is a Honda, then John's car is a Civic.
- [c] If John's car is a Civic, then John's car is a Honda.
- [d] John's car is a Honda and John's car is an Accord.
- [e] If John's car is a Honda, then John's car is an Accord.
- [f] If John's car is an Accord, then John's car is a Honda.
- [g] Every Civic is a Honda.
- [h] Every Honda is a Civic.

Now, which of the following are acceptable definitions for a limit ? HINT: The algebra is the same in every answer, but the English is different. They could all be acceptable, or they could all be unacceptable, or only one could be acceptable, or more than one could be acceptable.

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| <p>[a] <math>\lim_{x \rightarrow a} f(x) = L</math><br/>and <math>\varepsilon &gt; 0</math><br/>and a value <math>\delta &gt; 0</math><br/>and <math> f(x) - L  &lt; \varepsilon</math><br/>and <math>0 &lt;  x - a  &lt; \delta</math></p>                   | <p>[b] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>there exists a value <math>\delta &gt; 0</math> such that<br/>for all <math>\varepsilon &gt; 0</math><br/>if <math>0 &lt;  x - a  &lt; \delta</math><br/>then <math> f(x) - L  &lt; \varepsilon</math></p> |
| <p>[c] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>every <math>\varepsilon &gt; 0</math><br/>has some <math>\delta &gt; 0</math> where<br/>whenever <math>0 &lt;  x - a  &lt; \delta</math><br/>we have <math> f(x) - L  &lt; \varepsilon</math></p>  | <p>[d] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>for all <math>\varepsilon &gt; 0</math><br/>there exists a value <math>\delta &gt; 0</math> such that<br/>if <math> f(x) - L  &lt; \varepsilon</math><br/>then <math>0 &lt;  x - a  &lt; \delta</math></p> |
| <p>[e] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/><math>\varepsilon &gt; 0</math><br/>and a value <math>\delta &gt; 0</math> such that<br/><math>0 &lt;  x - a  &lt; \delta</math><br/>and <math> f(x) - L  &lt; \varepsilon</math></p>              | <p>[f] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>for all <math>\varepsilon &gt; 0</math> such that<br/>there exists a value <math>\delta &gt; 0</math><br/><math> f(x) - L  &lt; \varepsilon</math><br/>if <math>0 &lt;  x - a  &lt; \delta</math></p>      |
| <p>[g] <math>\lim_{x \rightarrow a} f(x) = L</math> such that<br/>for all <math>\varepsilon &gt; 0</math><br/>and a value <math>\delta &gt; 0</math><br/><math> f(x) - L  &lt; \varepsilon</math><br/>if <math>0 &lt;  x - a  &lt; \delta</math></p>          | <p>[h] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>for all <math>\varepsilon &gt; 0</math><br/>and a value <math>\delta &gt; 0</math><br/><math> f(x) - L  &lt; \varepsilon</math><br/>if <math>0 &lt;  x - a  &lt; \delta</math></p>                         |
| <p>[i] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>for all <math>\varepsilon &gt; 0</math><br/><math> f(x) - L  &lt; \varepsilon</math><br/>there exists a value <math>\delta &gt; 0</math> such that<br/><math>0 &lt;  x - a  &lt; \delta</math></p> | <p>[j] <math>\lim_{x \rightarrow a} f(x) = L</math> if<br/>there exists a value <math>\delta &gt; 0</math> such that<br/>for all <math>\varepsilon &gt; 0</math><br/><math> f(x) - L  &lt; \varepsilon</math><br/>if <math>0 &lt;  x - a  &lt; \delta</math></p>      |