THE PROPER WAY TO SHOW WORK WHEN FINDING CERTAIN LIMITS

When you are doing scratch work, you can use whatever shorthand you want as long as you understand it.
However, when you are writing up a solution which requires you to show work, you must write in proper mathematical notation.

INDETERMINATE LIMIT

Find $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$.

WRONG:

$$\frac{3^2-9}{3-3}=\frac{0}{0}=\lim_{x\to 3}\frac{(x+3)(x-3)}{x-3}=\lim_{x\to 3}(x+3)=6$$

WHY IT'S WRONG:

$\dfrac{3^2-9}{3-3}$ **and** $\dfrac{0}{0}$ **are not numbers, so they can't be equal to anything.**

RIGHT:

$$\lim_{x\to 3}\frac{(x+3)(x-3)}{x-3}=\lim_{x\to 3}(x+3)=6$$

scratch work: $\dfrac{3^2-9}{3-3}\to\dfrac{0}{0}$ indeterminate

LIMIT EQUAL TO INFINITY (OR NEGATIVE INFINITY)

Find $\lim_{x\to 3^+}\dfrac{x-4}{3-x}$.

WRONG:

$$\lim_{x\to 3^+}\frac{x-4}{3-x}=\frac{-1}{0^-}=\infty$$

WHY IT'S WRONG:

0^- **is not a number, so it can't be used in arithmetic expressions.**

Even if it could, $\dfrac{-1}{0^-}$ **is not a number, so it can't be equal to anything.**

RIGHT:

As $x\to 3^+$, $\quad x-4\to -1$

and $\quad 3-x\to 0^-$

So, $\lim_{x\to 3^+}\dfrac{x-4}{3-x}=\infty$

scratch work: $\dfrac{3^+-4}{3-3^+}\to\dfrac{-1}{0^-}\to\infty$

THE LIMITS FOR THE NUMERATOR AND DENOMINATOR ARE PLACED TO RESEMBLE A FRACTION (NUMERATOR ABOVE, DENOMINATOR BELOW) MAKING IT EASIER TO READ.

LIMIT AT INFINITY (OR NEGATIVE INFINITY)

Find $\lim_{x\to\infty}\frac{2x-4}{3-x}$.

WRONG:

$$\lim_{x\to\infty}\frac{2x-4}{3-x}=\frac{\infty-1}{3-\infty}=\frac{\infty}{-\infty}=-1$$

WHY IT'S WRONG:

∞ and $-\infty$ are not numbers, so they can't be used in arithmetic expressions.

Even if they could, $\frac{\infty-1}{3-\infty}$ and $\frac{\infty}{-\infty}$ are not numbers, so they can't be equal to anything.

Anway, the limit itself is wrong.

RIGHT:

$$\lim_{x\to\infty}\frac{\frac{1}{x}(2x-4)}{\frac{1}{x}(3-x)}=\lim_{x\to\infty}\frac{2-\frac{4}{x}}{\frac{3}{x}-1}=\frac{2}{-1}=-2$$

scratch work: $\frac{\infty-1}{3-\infty}\to\frac{\infty}{-\infty}$ indeterminate