Math 114 (8:30am – 9:20am)
Midterm 1 Version C
Wed Oct 19, 2011

NAME YOU ASKED TO BE CALLED: _____

SCORE: _____ / 140 POINTS

- **ALL PROBLEMS MUST BE SOLVED ALGEBRAICALLY TO EARN CREDIT (NO GUESS & CHECK)**
- **PUT A BOX AROUND EACH FINAL ANSWER**
- **SHOW COMPLETE AND PROPER WORK TO EARN FULL CREDIT**

Solve: The cost of paper needed to wrap a cylinder varies directly as the weight of the cylinder and inversely as the radius. A cylinder weighing 18 ounces with a radius of 2 inches requires 24 cents of paper to wrap. Find the cost of paper needed to wrap a cylinder with a radius of 4 inches that weighs 15 ounces.

SCORE: ___ / 15 POINTS

FOR FULL CREDIT, YOU MUST IDENTIFY WHAT ALL YOUR VARIABLES REPRESENT, FIND THE SPECIFIC EQUATION CONNECTING THEM, AND SUMMARIZE YOUR FINAL ANSWER IN A SENTENCE USING THE CORRECT UNITS OF MEASUREMENT.

C = cost of paper (cents)
w = weight of cylinder (ounces)
r = radius (inches)

$$C = \frac{kw}{r} \qquad C = \frac{8w}{3r}$$
$$24 = \frac{k(18)}{2} \qquad C = \frac{8(15)}{3(4)}$$
$$24 = 9k \qquad C = 10$$
$$\frac{8}{3} = k$$

It costs 10 cents to wrap the cylinder.

Subtract and simplify: $\dfrac{x+3}{x^2-1} - \dfrac{x+6}{x^2-3x-4}$

SCORE: ___ / 15 POINTS

$$= \frac{x+3}{(x+1)(x-1)} - \frac{x+6}{(x+1)(x-4)}$$
$$= \frac{x+3}{(x+1)(x-1)}\frac{x-4}{x-4} - \frac{x+6}{(x+1)(x-4)}\frac{x-1}{x-1}$$
$$= \frac{x^2-x-12-(x^2+5x-6)}{(x+1)(x-1)(x-4)}$$
$$= \frac{-6x-6}{(x+1)(x-1)(x-4)}$$
$$= \frac{-6(x+1)}{(x+1)(x-1)(x-4)}$$
$$= \boxed{\frac{-6}{(x+1)(x-4)}}$$

Solve: A number divided by twelve is equal to four divided by two less than that number. Find the number. **CHECK YOUR ANSWER(S).** **SCORE: ___ / 15 POINTS**

$$\frac{x}{12}=\frac{4}{x-2}$$

$$x^2-2x=48$$

$$x^2-2x-48=0$$

$$(x-8)(x+6)=0$$

$$\boxed{x=8 \text{ or } x=-6}$$

CHECK:

$x=8$: $\frac{8}{12}=\frac{2}{3}$ and $\frac{4}{6}=\frac{2}{3}$

$x=-6$: $\frac{-6}{12}=-\frac{1}{2}$ and $\frac{4}{-8}=-\frac{1}{2}$

Find the equation of the vertical asymptote of $y=\frac{5+11x}{17x-10}$. **SCORE: ___ / 8 POINTS**

$$17x-10=0$$

$$\boxed{x=\frac{10}{17}}$$

Find the equation of the horizontal asymptote of $y=\frac{5+11x}{17x-10}$.

$y\approx\frac{11x}{17x}$ for large values of x

$$\boxed{y=\frac{11}{17}}$$

Simplify: $\frac{2x^2+5x-3}{6x^2+13x-15}$ **SPECIFY ANY RESTRICTIONS**. **SCORE: ___ / 15 POINTS**

$$=\frac{(2x-1)(x+3)}{(6x-5)(x+3)}$$

$$=\boxed{\frac{2x-1}{6x-5}}$$

$x+3\neq 0$

$x\neq -3$

Solve for x: $\dfrac{3}{x^2-x-2}-\dfrac{4}{x^2-4}=\dfrac{1}{x+1}$ **CHECK YOUR ANSWER(S)** **SCORE: ___ / 15 POINTS**

$$(x-2)(x+1)(x+2)\left[\frac{3}{(x-2)(x+1)}-\frac{4}{(x-2)(x+2)}\right]=\frac{1}{x+1}(x-2)(x+1)(x+2)$$
$$3(x+2)-4(x+1)=(x-2)(x+2)$$
$$3x+6-4x-4=x^2-4$$
$$-x+2=x^2-4$$
$$0=x^2+x-6$$
$$0=(x+3)(x-2)$$
$$\boxed{x=-3} \text{ or } x=2$$

CHECK:

$x=-3$

$$\frac{3}{10}-\frac{4}{5}=\frac{3}{10}-\frac{8}{10}=-\frac{5}{10}=-\frac{1}{2}$$
$$\frac{1}{-2}=-\frac{1}{2}$$

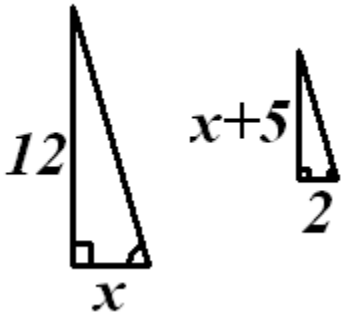
$x=2$

$\dfrac{3}{0}$ IS UNDEFINED

Simplify: $\dfrac{1-\dfrac{8}{x-6}}{\dfrac{2}{x-6}-\dfrac{3}{x-2}}$ **SCORE: ___ / 15 POINTS**

$$=\frac{1-\dfrac{8}{x-6}}{\dfrac{2}{x-6}-\dfrac{3}{x-2}}\cdot\frac{(x-6)(x-2)}{(x-6)(x-2)}$$
$$=\frac{(x-6)(x-2)-8(x-2)}{2(x-2)-3(x-6)}$$
$$=\frac{x^2-8x+12-8x+16}{2x-4-3x+18}$$
$$=\frac{x^2-16x+28}{-x+14}$$
$$=\frac{(x-14)(x-2)}{-(x-14)}$$
$$=\boxed{-(x-2) \text{ or } 2-x}$$

Solve for x in the following similar triangles: **SCORE: ___ / 12 POINTS**



$$\frac{12}{x} = \frac{x+5}{2}$$
$$24 = x^2 + 5x$$
$$0 = x^2 + 5x - 24$$
$$0 = (x+8)(x-3)$$
$$x = -8 \text{ or } \boxed{x = 3}$$

Divide and simplify: $\dfrac{27x^2 - 12}{50x^2 - 40x^3} \div \dfrac{27x + 18}{20x^2 - 25x}$ **SCORE: ___ / 15 POINTS**

$$= \frac{3(9x^2 - 4)}{-10x^2(4x - 5)} \div \frac{9(3x + 2)}{5x(4x - 5)}$$
$$= \frac{3(3x + 2)(3x - 2)}{-10x^2(4x - 5)} \times \frac{5x(4x - 5)}{9(3x + 2)}$$
$$= \frac{3x - 2}{-2x} \times \frac{1}{3}$$
$$= \frac{3x - 2}{-6x}$$
$$= \boxed{-\frac{(3x - 2)}{6x} \text{ or } \frac{2 - 3x}{6x}}$$

Subtract and simplify: $\dfrac{7x^2 - 9x - 5}{x^2 - x - 6} - \dfrac{5x^2 - 6x + 4}{x^2 - x - 6}$ **SCORE: ___ / 15 POINTS**

$$= \frac{2x^2 - 3x - 9}{x^2 - x - 6}$$
$$= \frac{(x - 3)(2x + 3)}{(x - 3)(x + 2)}$$
$$= \boxed{\frac{2x + 3}{x + 2}}$$