Math 114 (8:30am – 9:20am)
Midterm 1 Version R
Wed Oct 19, 2011

NAME YOU ASKED TO BE CALLED: _____

SCORE: _____ / 140 POINTS

- **ALL PROBLEMS MUST BE SOLVED ALGEBRAICALLY TO EARN CREDIT (NO GUESS & CHECK)**
- **PUT A BOX AROUND EACH FINAL ANSWER**
- **SHOW COMPLETE AND PROPER WORK TO EARN FULL CREDIT**

Solve: The cost of paper needed to wrap a cylinder varies directly as the weight of the cylinder and inversely as the radius. A cylinder weighing 15 ounces with a radius of 3 inches requires 12 cents of paper to wrap. Find the cost of paper needed to wrap a cylinder with a radius of 2 inches that weighs 20 ounces.

SCORE: ___ / 15 POINTS

FOR FULL CREDIT, YOU MUST IDENTIFY WHAT ALL YOUR VARIABLES REPRESENT, FIND THE SPECIFIC EQUATION CONNECTING THEM, AND SUMMARIZE YOUR FINAL ANSWER IN A SENTENCE USING THE CORRECT UNITS OF MEASUREMENT.

C = cost of paper (cents)
w = weight of cylinder (ounces)
r = radius (inches)

$$C = \frac{kw}{r} \qquad C = \frac{12w}{5r}$$
$$12 = \frac{k(15)}{3} \qquad C = \frac{12(20)}{5(2)}$$
$$12 = 5k \qquad C = 24$$
$$\frac{12}{5} = k$$

It costs 24 cents to wrap the cylinder.

Subtract and simplify: $\dfrac{x+1}{x^2-8x+15} - \dfrac{x+3}{x^2-9x+18}$

SCORE: ___ / 15 POINTS

$$= \frac{x+1}{(x-3)(x-5)} - \frac{x+3}{(x-3)(x-6)}$$
$$= \frac{x+1}{(x-3)(x-5)}\frac{x-6}{x-6} - \frac{x+3}{(x-3)(x-6)}\frac{x-5}{x-5}$$
$$= \frac{x^2-5x-6-(x^2-2x-15)}{(x-3)(x-5)(x-6)}$$
$$= \frac{-3x+9}{(x-3)(x-5)(x-6)}$$
$$= \frac{-3(x-3)}{(x-3)(x-5)(x-6)}$$
$$= \boxed{\frac{-3}{(x-5)(x-6)}}$$

Solve: A number divided by four is equal to fifteen divided by four less than that number. Find the number. **CHECK YOUR ANSWER(S).** **SCORE: ___ / 15 POINTS**

$$\frac{x}{4}=\frac{15}{x-4}$$

$$x^2-4x=60$$

$$x^2-4x-60=0$$

$$(x-10)(x+6)=0$$

$$\boxed{x=10 \text{ or } x=-6}$$

CHECK: $x=10$ $x=-6$

$$\frac{10}{4}=\frac{5}{2} \qquad \frac{-6}{4}=-\frac{3}{2}$$

$$\frac{15}{6}=\frac{5}{2} \qquad \frac{15}{-10}=-\frac{3}{2}$$

Find the equation of the vertical asymptote of $y=\frac{5+10x}{17x-11}$. **SCORE: ___ / 8 POINTS**

$$17x-11=0$$

$$\boxed{x=\frac{11}{17}}$$

Find the equation of the horizontal asymptote of $y=\frac{5+10x}{17x-11}$.

$y\approx\frac{10x}{17x}$ for large values of x

$$\boxed{y=\frac{10}{17}}$$

Simplify: $\frac{2x^2+5x-3}{6x^2+11x-21}$ **SPECIFY ANY RESTRICTIONS**. **SCORE: ___ / 15 POINTS**

$$=\frac{(2x-1)(x+3)}{(6x-7)(x+3)}$$

$$=\boxed{\frac{2x-1}{6x-7}} \qquad x+3\neq 0$$

$$x\neq -3$$

Solve for x: $\dfrac{3}{x^2-7x+10}-\dfrac{2}{x^2-8x+15}=\dfrac{1}{x-2}$ **CHECK YOUR ANSWER(S)** **SCORE: ___ / 15 POINTS**

$$(x-2)(x-5)(x-3)\left[\frac{3}{(x-2)(x-5)}-\frac{2}{(x-3)(x-5)}\right]=\frac{1}{x-2}(x-2)(x-5)(x-3)$$

$3(x-3)-2(x-2)=(x-5)(x-3)$

$3x-9-2x+4=x^2-8x+15$

$x-5=x^2-8x+15$

$0=x^2-9x+20$

$0=(x-4)(x-5)$

$\boxed{x=4}$ or $x=5$

CHECK:

$x=4$

$\dfrac{3}{-2}-\dfrac{2}{-1}=-\dfrac{3}{2}+2=\dfrac{1}{2}$

$\dfrac{1}{2}$

$x=5$

$\dfrac{3}{0}$ IS UNDEFINED

Simplify: $\dfrac{1-\dfrac{6}{x-5}}{\dfrac{3}{x-5}-\dfrac{4}{x-3}}$ **SCORE: ___ / 15 POINTS**

$$=\frac{1-\dfrac{6}{x-5}}{\dfrac{3}{x-5}-\dfrac{4}{x-3}}\cdot\frac{(x-5)(x-3)}{(x-5)(x-3)}$$

$$=\frac{(x-5)(x-3)-6(x-3)}{3(x-3)-4(x-5)}$$

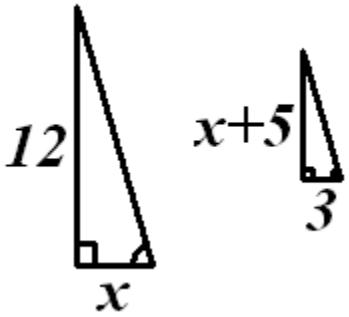
$$=\frac{x^2-8x+15-6x+18}{3x-9-4x+20}$$

$$=\frac{x^2-14x+33}{-x+11}$$

$$=\frac{(x-11)(x-3)}{-(x-11)}$$

$=\boxed{-(x-3) \text{ or } 3-x}$

Solve for x in the following similar triangles: **SCORE: ___ / 12 POINTS**



$$\frac{12}{x} = \frac{x+5}{3}$$
$$36 = x^2 + 5x$$
$$0 = x^2 + 5x - 36$$
$$0 = (x+9)(x-4)$$
$$x = -9 \text{ or } \boxed{x = 4}$$

Divide and simplify: $\dfrac{32x^2 - 18}{45x^2 - 30x^3} \div \dfrac{24x - 18}{10x^2 - 15x}$ **SCORE: ___ / 15 POINTS**

$$= \frac{2(16x^2 - 9)}{-15x^2(2x-3)} \div \frac{6(4x-3)}{5x(2x-3)}$$
$$= \frac{2(4x+3)(4x-3)}{-15x^2(2x-3)} \times \frac{5x(2x-3)}{6(4x-3)}$$
$$= \frac{4x+3}{-3x} \times \frac{1}{3}$$
$$= \frac{4x+3}{-9x}$$
$$= \boxed{-\frac{4x+3}{9x}}$$

Subtract and simplify: $\dfrac{7x^2 - 5x - 2}{x^2 + x - 2} - \dfrac{5x^2 - 6x + 1}{x^2 + x - 2}$ **SCORE: ___ / 15 POINTS**

$$= \frac{2x^2 + x - 3}{x^2 + x - 2}$$
$$= \frac{(x-1)(2x+3)}{(x-1)(x+2)}$$
$$= \boxed{\frac{2x+3}{x+2}}$$