How to rewrite the general formula for a sequence using index shifting

Suppose you have the sequence

$2^2, 3^2, 4^2, 5^2, 6^2, \ldots$

If the first term (2^2) is called a_2, then the formula is simply $a_n = n^2$.

Suppose you want to call the first term a_1 instead.
You can find the general formula using this new numbering
by performing a simple substitution on the original formula.

To simplify the reasoning,
we will call the sequence a and the index n, when using the original index,
and we will call the sequence b and the index k, when using the new index.

In other words,

For $n = 2$ and $k = 1$,	a_2	=	b_1	=	2^2
For $n = 3$ and $k = 2$,	a_3	=	b_2	=	3^2
For $n = 4$ and $k = 3$,	a_4	=	b_3	=	4^2
For $n = 5$ and $k = 4$,	a_5	=	b_4	=	5^2
For $n = 6$ and $k = 5$,	a_6	=	b_5	=	6^2

Notice that $a_n = b_k$ when $n = k + 1$.
Substituting $n = k + 1$ into $a_n = b_k$ gives $a_{k+1} = b_k$,
or in other words, $b_k = a_{k+1} = (k + 1)^2$ [by substituting $n = k + 1$ into $a_n = n^2$].
Since the name of the index is irrelevant, $b_n = (n + 1)^2$.
And since the name of the sequence is irrelevant, $a_n = (n + 1)^2$
when the index starts at $n = 1$.

Similarly, if you want to call the first term a_6 instead.
Temporarily rename the terms b_k instead of a_n.
So, the first term, which used to be called a_2 is now called b_6,
which means $n = k – 4$.
So, $b_k = a_n = a_{k-4} = (k – 4)^2$
So, $a_n = (n – 4)^2$ using the new index where the first term is called a_6.

Similar logic can be used to rewrite a summation.

For example, suppose you wanted to rewrite $\sum_{n=2}^{6} n^2$ as $\sum_{n=1}^{?} ?$.

First change the name of the index of summation in the new sum to get $\sum_{k=1}^{?} ?$.

Now, notice that $n = 2$ when $k = 1$, or in other words $n = k + 1$, or $k = n – 1$.
So the upper limit of summation $n = 6$ is equivalent to $k = 6 – 1 = 5$.
The formula n^2 is equivalent to $(k + 1)^2$.

So, the summation can be written as $\sum_{k=1}^{5} (k+1)^2$.

And since the name of index of summation is irrelevant,

the summation can be written as $\sum_{n=1}^{5} (n+1)^2$.