Math 22 (9:30am – 10:20am)
Midterm 1 Version O
Fri Oct 14, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 150 POINTS

RULES OF INFERENCE					
Contradiction (CONT)	$\sim p \to \mathbf{c}$ $\therefore p$				
Modus Ponens (MP)	$p \to q$ p $\therefore q$		**Modus Tollens (MT)**	$p \to q$ $\sim q$ $\therefore \sim p$	
Generalization (GEN)	p $\therefore p \vee q$	q $\therefore p \vee q$	**Specialization (SPEC)**	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	$p \vee q$ $\sim q$ $\therefore p$	**Conjunction (CONJ)**	p q $\therefore p \wedge q$	
Transitivity (TRAN)	$p \to q$ $q \to r$ $\therefore p \to r$		**Division into Cases (CASE)**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	

Use the rules of inference to prove that the following argument is valid. **SCORE: ___ / 20 POINTS**
For each statement or conclusion, write "GIVEN" if the statement was one of the original assumptions, or write the name of the rule of inference used. You may use the abbreviations shown in the table of rules of inference.

$\sim q \vee r \to \sim s$
$w \vee r$
$w \to s$
$p \wedge \sim q$
$\therefore r$

$p \wedge \sim q$ GIVEN
$\therefore \sim q$ SPEC
$\therefore \sim q \vee r$ GEN
$\sim q \vee r \to \sim s$ GIVEN
$\therefore \sim s$ MP
$w \to s$ GIVEN
$\therefore \sim w$ MT
$w \vee r$ GIVEN
$\therefore r$ ELIM

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 8 POINTS**

A RELATION F FROM SET A TO SET B IS A FUNCTION IF

$\forall x \in A, \exists y \in B : (x,y) \in F$ AND

$\forall x \in A, \forall y,z \in B, (x,y) \in F$ AND $(x,z) \in F \rightarrow y = z$

Fill in the blanks. **SCORE: ___ / 12 POINTS**

[a] In the conditional "$p \rightarrow q$",

p is called the HYPOTHESIS and q is called the CONCLUSION.

[b] The DISJUNCTION OF p AND q is denoted by $p \vee q$.

[c] The CARTESIAN PRODUCT OF SETS A AND B is denoted by $A \times B$.

[d] A is a **proper** subset of B if and only if EVERY ELEMENT OF A IS ALSO AN ELEMENT OF B, AND SOME ELEMENT OF B IS NOT AN ELEMENT OF A.

[e] The argument

"If you score more than 93%, then you will receive an A.

You did not score more than 93%.

Therefore, you will not receive an A."

is an example of INVERSE ERROR.

[f] We say an argument (or argument form) is valid if and only if IN ALL SITUATIONS WHERE ALL THE HYPOTHESES ARE TRUE, THE CONCLUSION MUST BE TRUE.

NOTE: Your answer should not involve the terms "truth table" or "critical row".

Determine whether the following argument is valid or invalid. Use a diagram to informally support your answer. **SCORE: ___ / 10 POINTS**

Some math classes are hard.
Some hard classes require a lot of work.
Therefore, some math classes require a lot of work.

INVALID

Consider the following statement which contains nonsense words (highlighted). **SCORE: ___ / 10 POINTS**

"If Pela is a **flura**, then Mola has a **griba**."

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write a logically equivalent statement using "necessary", without using "if".

MOLA HAVING A GRIBA IS NECESSARY FOR PELA TO BE A FLURA

[b] Write a logically equivalent statement using "only if".

PELA IS A FLURA ONLY IF MOLA HAS A GRIBA

[c] Write the inverse of the original statement.

IF PELA IS NOT A FLURA, THEN MOLA DOES NOT HAVE A GRIBA

Let $P(x)$ = "x is odd".
Let $Q(x)$ = "$x^2 > x + 1$".
Let D = {–5, –1, 0, 2, 3} be the domain of both predicates. **SCORE: ___ / 15 POINTS**

[a] Is the statement "$P(x) \Rightarrow Q(x)$" true or false ? **Justify your answer.**

TRUTH SET OF $P = \{-5, -1, 3\}$
$\subseteq$ TRUTH SET OF $Q = \{-5, -1, 2, 3\}$

TRUE

[b] Is the statement "$P(x) \Leftrightarrow Q(x)$" true or false ? **Justify your answer.**

TRUTH SETS ARE NOT EQUAL
FALSE

Determine if the following statement is a tautology, a contradiction or neither. **SCORE: ___ / 15 POINTS**

$\sim p \leftrightarrow (p \oplus \sim q)$

p	q	$\sim q$	$p \oplus \sim q$	$\sim p$	$\sim p \leftrightarrow (p \oplus \sim q)$
T	T	F	T	F	F
T	F	T	F	F	T
F	T	F	F	T	F
F	F	T	T	T	T

NEITHER

Consider the statement "The square root of a prime number is always irrational." **SCORE: ___ / 20 POINTS**

[a] Write the statement symbolically using **exactly one quantifier** and **no conditionals**.
Identify your domain and predicate clearly.

$\forall x \in D,\ P(x)$

$D = \{\text{PRIME NUMBERS}\}$

$P(x) = "\sqrt{x} \text{ IS IRRATIONAL}"$

[b] Write the statement symbolically using **exactly one quantifier** and **one conditional**.
Identify your domain and predicates clearly.

$\forall x \in D,\ P(x) \rightarrow Q(x)$ $D = \{\text{NUMBERS}\}$

$P(x) = "x \text{ IS PRIME}"$

$Q(x) = "\sqrt{x} \text{ IS IRRATIONAL}"$

[c] Write the contrapositive of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

$\forall x \in D,\ \sim Q(x) \rightarrow \sim P(x)$

IF THE SQUARE ROOT OF A NUMBER IS NOT IRRATIONAL, THAT NUMBER IS NOT PRIME

[d] Write the negation of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

$\exists x \in D : P(x) \wedge \sim Q(x)$

THERE IS A PRIME NUMBER WHOSE SQUARE ROOT IS NOT IRRATIONAL.

Use truth tables to determine if the following argument is valid. Mark all critical rows clearly. **SCORE: ___ / 20 POINTS**

If Anh enrolled in Math 22, then Anh is a computer science major or Anh is a math major.
If Anh is a math major, then Anh is a computer science major.
Therefore, Anh is a computer science major or Anh is not enrolled in Math 22.

$p \rightarrow (q \vee r)$

$r \rightarrow q$

$\therefore q \vee \sim p$

VALID

p	q	r	q∨r	p→(q∨r)	r→q	~p	q∨~p
T	T	T	T	T	T	F	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	F	F
T	F	F	F	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	T	T	T

The following table shows the cargo that was delivered by 3 drivers during 3 months. **SCORE: ___ / 20 POINTS**

	June	July	August
Alan	Appliances	Appliances & Machinery	
Barb	Appliances & Furniture	Machinery	
Carl		Appliances	Furniture

NOTE: Alan & Barb did not deliver any cargo during August, and Carl did not deliver any cargo during June.

Let D = set of drivers = {Alan, Barb, Carl}.
Let M = set of months = {June, July, August}.
Let C = set of cargo = {appliances, furniture, machinery}.

[a] Write the negation of "$\exists c \in C$: $\forall m \in M$, Alan delivered c during m". You may use quantifiers and variables in your answer.

$\forall c \in C, \exists m \in M$: ALAN DID NOT DELIVER C DURING m

[b] Determine if the following statements are true or false. **Justify your answers with examples and/or counterexamples.**

[i] $\forall d \in D$, $\exists m \in M$: d delivered appliances during m

d = ALAN $\exists m \in M$: ALAN DELIVERED APPLIANCES DURING m TRUE (JUNE/JULY)

d = BARB " BARB " TRUE (JUNE)

d = CARL " CARL " TRUE (JULY)

TRUE

[ii] $\exists d \in D$: $\forall m \in M$, d made a delivery during m

d = ALAN $\forall m \in M$, ALAN MADE A DELIVERY DURING m FALSE (AUG)

d = BARB " BARB " FALSE (AUG)

d = CARL " CARL " FALSE (JUNE)

FALSE

OPTIONAL BONUS QUESTIONS ON OTHER SIDE

Math 22 (9:30am – 10:20am)
Midterm 1 Version P
Fri Oct 14, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 150 POINTS

RULES OF INFERENCE					
Modus Ponens (MP)	$p \to q$ p $\therefore q$		**Contradiction (CONT)**	$\sim p \to \mathbf{c}$ $\therefore p$	
Modus Tollens (MT)	$p \to q$ $\sim q$ $\therefore \sim p$				
Generalization (GEN)	p $\therefore p \vee q$	q $\therefore p \vee q$	**Specialization (SPEC)**	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	$p \vee q$ $\sim q$ $\therefore p$	**Conjunction (CONJ)**	p q $\therefore p \wedge q$	
Transitivity (TRAN)	$p \to q$ $q \to r$ $\therefore p \to r$		**Division into Cases (CASE)**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	

Use the rules of inference to prove that the following argument is valid. **SCORE: ___ / 20 POINTS**
For each statement or conclusion, write "GIVEN" if the statement was one of the original assumptions, or write the name of the rule of inference used. You may use the abbreviations shown in the table of rules of inference.

$\sim r \vee s \to \sim w$
$p \vee s$
$p \to w$
$q \wedge \sim r$
$\therefore s$

$q \wedge \sim r$	GIVEN
$\therefore \sim r$	SPEC
$\therefore \sim r \vee s$	GEN
$\sim r \vee s \to \sim w$	GIVEN
$\therefore \sim w$	MP
$p \to w$	GIVEN
$\therefore \sim p$	MT
$p \vee s$	GIVEN
$\therefore s$	ELIM

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 8 POINTS**

SEE VERSION O

Fill in the blanks. SEE VERSION O **SCORE: ___ / 12 POINTS**

[a] In the conditional "$p \to q$",

p is called the _____ and q is called the _____.

[b] The _____ is denoted by $p \vee q$.

[c] The _____ is denoted by $A \times B$.

[d] A is a **proper** subset of B if and only if _____

_____

_____.

[e] The argument

"If you score more than 93%, then you will receive an A.

You did not score more than 93%.

Therefore, you will not receive an A."

is an example of _____.

[f] We say an argument (or argument form) is valid if and only if _____

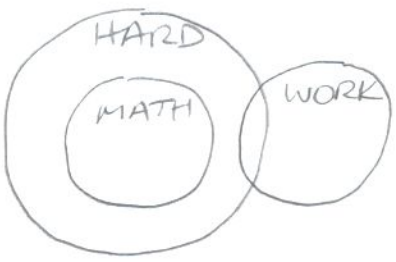
_____

_____.

NOTE: Your answer should not involve the terms "truth table" or "critical row".

Determine whether the following argument is valid or invalid. Use a diagram to informally support your answer. **SCORE: ___ / 10 POINTS**

All math classes are hard.
Some hard classes require a lot of work.
Therefore, some math classes require a lot of work. INVALID



Consider the following statement which contains nonsense words (highlighted). **SCORE: ___ / 10 POINTS**

"If Pela has a **griba**, then Mola is a **flura**."

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write a logically equivalent statement using "necessary", without using "if".

MOLA BEING A FLURA IS NECESSARY FOR PELA TO HAVE A GRIBA

[b] Write a logically equivalent statement using "only if".

PELA HAS A GRIBA ONLY IF MOLA IS A FLURA

[c] Write the inverse of the original statement.

IF PELA DOES NOT HAVE A GRIBA, THEN MOLA IS NOT A FLURA

Let $P(x)$ = "x is odd".
Let $Q(x)$ = "$x^2 > x + 1$".
Let D = {–5, –1, 0, 2, 3} be the domain of both predicates. **SCORE: ___ / 15 POINTS**

[a] Is the statement "$P(x) \Rightarrow Q(x)$" true or false ? **Justify your answer.**

SEE VERSION O

[b] Is the statement "$P(x) \Leftrightarrow Q(x)$" true or false ? **Justify your answer.**

Determine if the following statement is a tautology, a contradiction or neither. **SCORE: ___ / 15 POINTS**

$\sim q \leftrightarrow (\sim p \oplus q)$

p	q	$\sim p$	$\sim p \oplus q$	$\sim q$	$\sim q \leftrightarrow (\sim p \oplus q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	F	T
F	F	T	T	T	T

NEITHER

Consider the statement "The square root of a prime number is always irrational." **SCORE: ___ / 20 POINTS**

[a] Write the statement symbolically using **exactly one quantifier** and **no conditionals**.
Identify your domain and predicate clearly.

SEE VERSION C

[b] Write the statement symbolically using **exactly one quantifier** and **one conditional**.
Identify your domain and predicates clearly.

[c] Write the contrapositive of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

[d] Write the negation of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

Use truth tables to determine if the following argument is valid. Mark all critical rows clearly. **SCORE: ___ / 20 POINTS**

If Anh enrolled in Math 22, then Anh is a computer science major or Anh is a math major.
If Anh is a math major, then Anh is a computer science major.
Therefore, Anh is a computer science major or Anh is not enrolled in Math 22.

SEE VERSION C

The following table shows the cargo that was delivered by 3 drivers during 3 months.

SCORE: ___ / 20 POINTS

	June	July	August
Alan	Appliances	Appliances & Machinery	
Barb	Appliances & Furniture	Machinery	
Carl		Appliances	Furniture

NOTE: Alan & Barb did not deliver any cargo during August, and Carl did not deliver any cargo during June.

Let D = set of drivers = {Alan, Barb, Carl}.
Let M = set of months = {June, July, August}.
Let C = set of cargo = {appliances, furniture, machinery}.

[a] Write the negation of "$\exists c \in C$: $\forall m \in M$, Alan delivered c during m". You may use quantifiers and variables in your answer.

$\forall c \in C, \exists m \in M$: ALAN DID NOT DELIVER C DURING m

[b] Determine if the following statements are true or false. **Justify your answers with examples and/or counterexamples.**

[i] $\forall c \in C$, $\exists m \in M$: Barb delivered c during m

C = APP $\exists m \in M$: BARB DELIVERED APP DURING m
TRUE (JUNE)

C = FURN " " FURN "
TRUE (JUNE)

C = MACH " " MACH "
TRUE (JULY)

TRUE

[ii] $\exists m \in M$: $\forall d \in D$, d delivered appliances during m

m = JUNE $\forall d \in D$, d DELIVERED APP DURING JUNE
FALSE (CARL)

m = JULY " " JULY
FALSE (BARB)

m = AUG " " AUG
FALSE (ALL)

FALSE

OPTIONAL BONUS QUESTIONS ON OTHER SIDE

Math 22 (9:30am – 10:20am)
Midterm 1 Version E
Fri Oct 14, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 150 POINTS

RULES OF INFERENCE					
			Contradiction (CONT)	$\sim p \to \mathbf{c}$ $\therefore p$	
Modus Ponens (MP)	$p \to q$ p $\therefore q$		**Modus Tollens (MT)**	$p \to q$ $\sim q$ $\therefore \sim p$	
Generalization (GEN)	p $\therefore p \vee q$	q $\therefore p \vee q$	**Specialization (SPEC)**	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	$p \vee q$ $\sim q$ $\therefore p$	**Conjunction (CONJ)**	p q $\therefore p \wedge q$	
Transitivity (TRAN)	$p \to q$ $q \to r$ $\therefore p \to r$		**Division into Cases (CASE)**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	

Use the rules of inference to prove that the following argument is valid. **SCORE: ___ / 20 POINTS**
For each statement or conclusion, write "GIVEN" if the statement was one of the original assumptions, or write the name of the rule of inference used. You may use the abbreviations shown in the table of rules of inference.

$\sim s \vee w \to \sim p$
$q \vee w$
$q \to p$
$r \wedge \sim s$
$\therefore w$

$r \wedge \sim s$	GIVEN
$\therefore \sim s$	SPEC
$\therefore \sim s \vee w$	GEN
$\sim s \vee w \to \sim p$	GIVEN
$\therefore \sim p$	MP
$q \to p$	GIVEN
$\therefore \sim q$	MT
$q \vee w$	GIVEN
$\therefore w$	ELIM

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 8 POINTS**

SEE VERSION O

Fill in the blanks. SEE VERSION O **SCORE: ___ / 12 POINTS**

[a] In the conditional "$p \to q$",

p is called the _____ and q is called the _____ .

[b] The _____ is denoted by $p \vee q$.

[c] The _____ is denoted by $A \times B$.

[d] A is a **proper** subset of B if and only if _____

_____

_____.

[e] The argument

"If you score more than 93%, then you will receive an A.

You did not score more than 93%.

Therefore, you will not receive an A."

is an example of _____.

[f] We say an argument (or argument form) is valid if and only if _____

_____

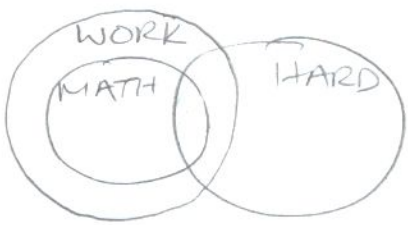
_____.

NOTE: Your answer should not involve the terms "truth table" or "critical row".

Determine whether the following argument is valid or invalid. Use a diagram to informally support your answer. **SCORE: ___ / 10 POINTS**

Some math classes are hard.
All math classes require a lot of work.
Therefore, some hard classes require a lot of work.

VALID



Consider the following statement which contains nonsense words (highlighted). **SCORE: ___ / 10 POINTS**

"If Mola is a **flura**, then Pela has a **griba**."

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write a logically equivalent statement using "necessary", without using "if".

PELA HAVING A GRIBA IS NECESSARY FOR MOLA TO BE A FLURA

[b] Write a logically equivalent statement using "only if".

MOLA IS A FLURA ONLY IF PELA HAS A GRIBA

[c] Write the inverse of the original statement.

IF MOLA IS NOT A FLURA, THEN PELA DOESNOT HAVE A GRIBA

Let $P(x)$ = "x is odd".
Let $Q(x)$ = "$x^2 > x + 1$".
Let D = {–5, –1, 0, 2, 3} be the domain of both predicates. **SCORE: ___ / 15 POINTS**

[a] Is the statement "$P(x) \Rightarrow Q(x)$" true or false ? **Justify your answer.**

SEE VERSION O

[b] Is the statement "$P(x) \Leftrightarrow Q(x)$" true or false ? **Justify your answer.**

Determine if the following statement is a tautology, a contradiction or neither. **SCORE: ___ / 15 POINTS**

$p \leftrightarrow (\sim p \oplus \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \oplus \sim q$	$p \leftrightarrow (\sim p \oplus \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	F	T

NEITHER

Consider the statement "The square root of a negative number is always imaginary." **SCORE: ___ / 20 POINTS**

[a] Write the statement symbolically using **exactly one quantifier** and **no conditionals**.
Identify your domain and predicate clearly.

$\forall x \in D, P(x)$

$D = \{\text{NEGATIVE NUMBERS}\}$

$P(x) = "\sqrt{x} \text{ IS IMAGINARY}"$

[b] Write the statement symbolically using **exactly one quantifier** and **one conditional**.
Identify your domain and predicates clearly.

$\forall x \in D, P(x) \rightarrow Q(x)$ $D = \{\text{NUMBERS}\}$

$P(x) = "x < 0"$

$Q(x) = "\sqrt{x} \text{ IS IMAGINARY}"$

[c] Write the contrapositive of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

$\forall x \in D, \sim Q(x) \rightarrow \sim P(x)$

IF THE SQUARE ROOT OF A NUMBER IS NOT IMAGINARY, THAT NUMBER IS NOT NEGATIVE

[d] Write the negation of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

$\exists x \in D : P(x) \wedge \sim Q(x)$

THERE IS A NEGATIVE NUMBER WHOSE SQUARE ROOT IS NOT IMAGINARY

Use truth tables to determine if the following argument is valid. Mark all critical rows clearly. **SCORE: ___ / 20 POINTS**

If Anh enrolled in Math 22, then Anh is a computer science major or Anh is a math major.
If Anh is a math major, then Anh is a computer science major.
Therefore, Anh is a computer science major or Anh is not enrolled in Math 22.

SEE VERSION ○

The following table shows the cargo that was delivered by 3 drivers during 3 months. **SCORE: ___ / 20 POINTS**

	June	July	August
Alan	Appliances	Appliances & Machinery	
Barb	Appliances & Furniture	Machinery	
Carl		Appliances	Furniture

NOTE: Alan & Barb did not deliver any cargo during August, and Carl did not deliver any cargo during June.

Let D = set of drivers = {Alan, Barb, Carl}.
Let M = set of months = {June, July, August}.
Let C = set of cargo = {appliances, furniture, machinery}.

[a] Write the negation of "$\exists c \in C$: $\forall m \in M$, Alan delivered c during m". You may use quantifiers and variables in your answer.

$\forall c \in C, \exists m \in M$: ALAN DID NOT DELIVER C DURING m

[b] Determine if the following statements are true or false. **Justify your answers with examples and/or counterexamples.**

[i] $\forall m \in M$, $\exists d \in D$: d delivered appliances during m

m = JUNE $\exists d \in D$: d DELIVERED APPLIANCES DURING JUNE
TRUE (ALAN/BARB)

m = JULY " " JULY
TRUE (ALAN/CARL)

m = AUG " " AUG
FALSE

FALSE

[ii] $\exists d \in D$: $\forall c \in C$, d delivered c at some time during the period from June to August

d = ALAN $\forall c \in C$, ALAN DELIVERED c SOME TIME ...
FALSE (FURN)

d = BARB $\forall c \in C$, BARB DELIVERED c SOME TIME ...
TRUE
(APP: JUNE
FURN: JUNE
MACH: JULY)

TRUE

OPTIONAL BONUS QUESTIONS ON OTHER SIDE

Math 22 (9:30am – 10:20am)
Midterm 1 Version N
Fri Oct 14, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 150 POINTS

RULES OF INFERENCE					
Modus Ponens (MP)	$p \to q$ p $\therefore q$		**Contradiction (CONT)**	$\sim p \to \mathbf{c}$ $\therefore p$	
Generalization (GEN)	p $\therefore p \vee q$	q $\therefore p \vee q$	**Specialization (SPEC)**	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	$p \vee q$ $\sim q$ $\therefore p$	**Conjunction (CONJ)**	p q $\therefore p \wedge q$	
Transitivity (TRAN)	$p \to q$ $q \to r$ $\therefore p \to r$		**Division into Cases (CASE)**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	

Use the rules of inference to prove that the following argument is valid. **SCORE: ___ / 20 POINTS**
For each statement or conclusion, write "GIVEN" if the statement was one of the original assumptions, or write the name of the rule of inference used. You may use the abbreviations shown in the table of rules of inference.

$\sim w \vee p \to \sim q$
$r \vee p$
$r \to q$
$s \wedge \sim w$
$\therefore p$

$s \wedge \sim w$	GIVEN
$\therefore \sim w$	SPEC
$\therefore \sim w \vee p$	GEN
$\sim w \vee p \to \sim q$	GIVEN
$\therefore \sim q$	MP
$r \to q$	GIVEN
$\therefore \sim r$	MT
$r \vee p$	GIVEN
$\therefore p$	ELIM

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 8 POINTS**

SEE VERSION O

Fill in the blanks. SEE VERSION O **SCORE: ___ / 12 POINTS**

[a] In the conditional "$p \to q$",

p is called the _____ and q is called the _____.

[b] The _____ is denoted by $p \vee q$.

[c] The _____ is denoted by $A \times B$.

[d] A is a **proper** subset of B if and only if _____

_____

_____.

[e] The argument

"If you score more than 93%, then you will receive an A.

You did not score more than 93%.

Therefore, you will not receive an A."

is an example of _____.

[f] We say an argument (or argument form) is valid if and only if _____

_____

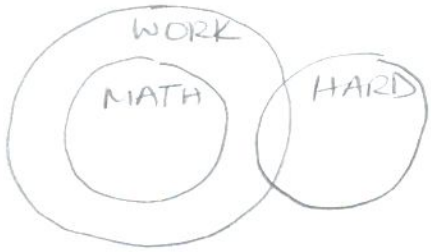
_____.

NOTE: Your answer should not involve the terms "truth table" or "critical row".

Determine whether the following argument is valid or invalid. Use a diagram to informally support your answer. **SCORE: ___ / 10 POINTS**

All math classes require a lot of work.
Some hard classes require a lot of work.
Therefore, some math classes are hard.

INVALID



Consider the following statement which contains nonsense words (highlighted). **SCORE: ___ / 10 POINTS**

"If Mola has a **griba**, then Pela is a **flura**."

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write a logically equivalent statement using "necessary", without using "if".

PELA BEING A FLURA IS NECESSARY FOR MOLA TO HAVE A GRIBA

[b] Write a logically equivalent statement using "only if".

MOLA HAS A GRIBA ONLY IF PELA IS A FLURA

[c] Write the inverse of the original statement.

IF MOLA DOES NOT HAVE A GRIBA, THEN PELA IS NOT A FLURA

Let $P(x)$ = "x is odd". **SCORE: ___ / 15 POINTS**
Let $Q(x)$ = "$x^2 > x + 1$".
Let D = {–5, –1, 0, 2, 3} be the domain of both predicates.

[a] Is the statement "$P(x) \Rightarrow Q(x)$" true or false ? **Justify your answer.**

SEE VERSION O

[b] Is the statement "$P(x) \Leftrightarrow Q(x)$" true or false ? **Justify your answer.**

Determine if the following statement is a tautology, a contradiction or neither. **SCORE: ___ / 15 POINTS**

$q \leftrightarrow (\sim p \oplus \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \oplus \sim q$	$q \leftrightarrow (\sim p \oplus \sim q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	F	T

NEITHER

Consider the statement "The square root of a negative number is always imaginary." **SCORE: ___ / 20 POINTS**

[a] Write the statement symbolically using **exactly one quantifier** and **no conditionals**.
Identify your domain and predicate clearly.

SEE VERSION E

[b] Write the statement symbolically using **exactly one quantifier** and **one conditional**.
Identify your domain and predicates clearly.

[c] Write the contrapositive of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

[d] Write the negation of your answer in [b] symbolically,
and also as an English sentence without using any symbols or variables.

Use truth tables to determine if the following argument is valid. Mark all critical rows clearly. **SCORE: ___ / 20 POINTS**

If Anh enrolled in Math 22, then Anh is a computer science major or Anh is a math major.
If Anh is a math major, then Anh is a computer science major.
Therefore, Anh is a computer science major or Anh is not enrolled in Math 22.

SEE VERSION O

The following table shows the cargo that was delivered by 3 drivers during 3 months. **SCORE: ___ / 20 POINTS**

	June	July	August
Alan	Appliances	Appliances & Machinery	
Barb	Appliances & Furniture	Machinery	
Carl		Appliances	Furniture

NOTE: Alan & Barb did not deliver any cargo during August, and Carl did not deliver any cargo during June.

Let D = set of drivers = {Alan, Barb, Carl}.
Let M = set of months = {June, July, August}.
Let C = set of cargo = {appliances, furniture, machinery}.

[a] Write the negation of "$\exists c \in C$: $\forall m \in M$, Alan delivered c during m". You may use quantifiers and variables in your answer.

$\forall c \in C, \exists m \in M$: ALAN DID NOT DELIVER C DURING m

[b] Determine if the following statements are true or false. **Justify your answers with examples and/or counterexamples.**

[i] $\forall m \in M$, $\exists c \in C$: Barb delivered c during m

m = JUNE $\exists c \in C$: BARB DELIVERED C DURING JUNE
TRUE (APP/FURN)

m = JULY " " " JULY
TRUE (MACH)

m = AUG " " " AUG
FALSE

FALSE

[ii] $\exists c \in C$: $\forall d \in D$, d delivered c at some time during the period from June to August

C = APP $\forall d \in D$, d DELIVERED APP SOMETIME ...
TRUE
(ALAN: JUNE/JULY
BARB: JUNE
CARL: JULY)

TRUE

OPTIONAL BONUS QUESTIONS ON OTHER SIDE