Math 22 (9:30am – 10:20am)
Midterm 3
Tue Dec 6, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ____________

SCORE: ___ / 150 POINTS

Use an **element argument** to write a formal proof for the statement below. **SCORE: ___ / 20 POINTS**

For all sets A and B, $(A \cap B) \cup (A \cap B^C) = A$

DO NOT USE ANY SET IDENTITIES IN YOUR PROOF

SEE SECTION 6.2 #8

Find the mistake in the following "proof". **SCORE: ___ / 7 POINTS**

"Theorem": For all sets A and B, $A^C \cup B^C \subseteq (A \cup B)^C$

"Proof": Suppose A and B are subsets of a universal set U.
Let $x \in A^C \cup B^C$.
By definition of union, $x \in A^C$ or $x \in B^C$.
By definition of complement, $x \notin A$ or $x \notin B$.
By definition of union, $x \notin A \cup B$.
By definition of complement, $x \in (A \cup B)^C$.
So, $A^C \cup B^C \subseteq (A \cup B)^C$.

NOTE: The "theorem" is a false statement. What step in the "proof" is "justified" incorrectly ?

SEE SECTION 6.2 #21

Determine if the following statement is true or false. **SCORE: ___ / ? POINTS**
If it is true, write a formal proof. If it is false, find a counterexample.

For all sets A and B, $\wp(A \cup B) \subseteq \wp(A) \cup \wp(B)$

SEE SECTION 6.3 #18

Determine if the following statement is true or false. **SCORE: ___ / ? POINTS**
If it is true, write a formal proof. If it is false, find a counterexample.

If $f : R \to R$ and $g : R \to R$ are both onto,
and $(f + g) : R \to R$ is defined by $(f + g)(x) = f(x) + g(x)$ for all $x \in R$,
then $f + g$ is onto

SEE SECTION 7.2 #31

Let B be a Boolean algebra with operations $+$ and $\cdot$. **SCORE: ___ / 15 POINTS**
Prove the following statement using only the definition of a Boolean algebra, the Uniqueness of the Complement Law, the Uniqueness of 0 and 1, the Double Complement Law and the Idempotent Law.
<u>Give the names of the laws that justify each step of your proof.</u>

For all $a \in B$, $\quad a \cdot 0 = 0$

SEE SECTION 6.4 #4

Write a formal proof for the following statement. **SCORE: ___ / 15 POINTS**

If X and Y are sets and $F : X \to Y$ is a one-to-one correspondence, then $F^{-1} : Y \to X$ is one-to-one

SEE SECTION 7.2 THEOREM 7.2.3

There are 3 roads from city A to city B, and 5 roads from city B to city C. **SCORE: ___ / 8 POINTS**
How many different routes are there from A to B to C to B to A in which no road is traversed twice ?

SEE SECTION 9.2 #9c

A group of 8 people are attending the movies together. **SCORE: ___ / 10 POINTS**
Two of the people do not like each other and do not want to sit side-by-side. How many ways can the eight be seated together in a row ?

SEE SECTION 9.3 #12b

A student council consists of 15 students. Two council members always insist on serving on committees together. **SCORE: ___ / 10 POINTS**
If they can't serve together, they won't serve at all. How many ways can a committee of 6 be selected from the council membership ?

SEE SECTION 9.5 #6c

How many 3 digit code numbers contain one digit that occurs twice, and one other (different) digit ? **SCORE: ___ / 15 POINTS**
<u>NOTE: A code number can begin with 0 or 00.</u>

SEE COMBINATORICS HANDOUT ON MY WEBSITE

Let $m, n, k \in Z^+$ such that $m < n$. How many integers from m to n inclusive are divisible by k ? **SCORE: ___ / 8 POINTS**
ie. how many integers in the list $m, m+1, m+2, \cdots, n$ are divisible by k ?

$n \text{ div } k - (m-1) \text{ div } k$

Determine if the relation D defined below is an equivalence relation. **SCORE: ___ / 20 POINTS**
For each property of an equivalence relation, if that property is satisfied by D, write a formal proof, and if that property is not satisfied by D, find a counterexample.

For all $x, y \in R$, $\qquad xDy \Leftrightarrow xy \geq 0$

SEE SECTION 8.2 #11