Math 22 (9:30am – 10:20am)
Pop Quiz 4 Version M
Thu Oct 27, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 6 POINTS

Consider the true statement "If $x \bmod 6 = 4$, then $(-x) \bmod 6 = 2$". **SCORE: ___ / 1 POINT**
Find the **FIRST** error in the following incorrect "proof", and explain in **10 WORDS OR FEWER** what the error is.

"Proof": Let x be a particular but arbitrary integer such that $x \bmod 6 = 4$.
By the definition of mod, $x = 6k + 4$ for some integer k.
$\boxed{\text{If } (-x) \bmod 6 = 2}$, then by the definition of mod, $-x = 6q + 2$ for some integer q.
But $-x = -(6k + 4) = -6k - 4$.
So, $6q + 2 = -6k - 4$.
So, $6q + 6k = -6$.
So, $q + k = -1$.
$q + k$ is an integer by the closure of integers under addition, and -1 is an integer, so $q + k$ could equal -1.
So, $(-x) \bmod 6 = 2$.

The writer assumed what was to be proved was already true.

Consider the true statement "The square root of an irrational number is irrational". **SCORE: ___ / 1 POINT**
Write **ONLY THE FIRST SENTENCE** of a proof by contradiction.

"Assume there is an irrational number whose square root is not irrational (ie. rational)."

Prove the following statement. **SCORE: ___ / 4 POINTS**

DO NOT USE ANY THEOREMS OR EXERCISES FROM THE BOOK AS JUSTIFICATION.

The difference of any irrational number minus any rational number is irrational.

Proof by contradiction:

Assume the negation of the statement, that there exists an irrational number x, and a rational number y, such that $x - y$ is rational.

By definition of rational, $y = \frac{a}{b}$ and $x - y = \frac{c}{d}$ for some integers a, b, c, d such that $b \neq 0$ and $d \neq 0$.

So $x = y + (x - y) = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
where $ad + bc$ and bd are integers by the closure of integers under multiplication and addition,
and $bd \neq 0$ by the zero product property.
Therefore, by definition of rational, x is rational, which contradicts the assumption that x is irrational.
By contradiction, the negation is false, and the original statement is true.

Math 22 (9:30am – 10:20am)
Pop Quiz 4 Version A
Thu Oct 27, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 6 POINTS

Consider the true statement "If $x \bmod 6 = 4$, then $(-x) \bmod 6 = 2$". **SCORE: ___ / 1 POINT**
Find the **FIRST** error in the following incorrect "proof", and explain in **10 WORDS OR FEWER** what the error is.

"Proof": Let x be a particular but arbitrary integer such that $x \bmod 6 = 4$.
By the definition of mod, $x = 6k + 4$ for some integer k.
$\boxed{\text{If } (-x) \bmod 6 = 2}$, then by the definition of mod, $-x = 6q + 2$ for some integer q.
But $-x = -(6k + 4) = -6k - 4$.
So, $6q + 2 = -6k - 4$.
So, $6q + 6k = -6$.
So, $q + k = -1$.
$q + k$ is an integer by the closure of integers under addition, and -1 is an integer, so $q + k$ could equal -1.
So, $(-x) \bmod 6 = 2$.

The writer assumed what was to be proved was already true.

Consider the true statement "The square root of an irrational number is irrational". **SCORE: ___ / 1 POINT**
Write **ONLY THE FIRST SENTENCE** of a proof by contradiction.

"Assume there is an irrational number whose square root is not irrational (ie. rational)."

Prove the following statement. **SCORE: ___ / 4 POINTS**

DO NOT USE ANY THEOREMS OR EXERCISES FROM THE BOOK AS JUSTIFICATION.

The difference of any irrational number minus any rational number is irrational.

Proof by contradiction:

Assume the negation of the statement, that there exists an irrational number x, and a rational number y, such that $x - y$ is rational.

By definition of rational, $y = \frac{a}{b}$ and $x - y = \frac{c}{d}$ for some integers a, b, c, d such that $b \neq 0$ and $d \neq 0$.

So $x = y + (x - y) = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
where $ad + bc$ and bd are integers by the closure of integers under multiplication and addition,
and $bd \neq 0$ by the zero product property.
Therefore, by definition of rational, x is rational, which contradicts the assumption that x is irrational.
By contradiction, the negation is false, and the original statement is true.

Math 22 (9:30am – 10:20am)
Pop Quiz 4 Version R
Thu Oct 27, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 6 POINTS

Consider the true statement "If $x \bmod 6 = 4$, then $(-x) \bmod 6 = 2$".

SCORE: ___ / 1 POINT

Find the **FIRST** error in the following incorrect "proof", and explain in **10 WORDS OR FEWER** what the error is.

"Proof": Let x be a particular but arbitrary integer such that $x \bmod 6 = 4$.

By the definition of mod, $x = 6k + 4$ for some integer k.

If $(-x) \bmod 6 = 2$, then by the definition of mod, $-x = 6q + 2$ for some integer q.

But $-x = -(6k + 4) = -6k - 4$.

So, $6q + 2 = -6k - 4$.

So, $6q + 6k = -6$.

So, $q + k = -1$.

$q + k$ is an integer by the closure of integers under addition, and -1 is an integer, so $q + k$ could equal -1.

So, $(-x) \bmod 6 = 2$.

The writer assumed what was to be proved was already true.

Consider the true statement "The square root of an irrational number is irrational".

SCORE: ___ / 1 POINT

Write **ONLY THE FIRST SENTENCE** of a proof by contradiction.

"Assume there is an irrational number whose square root is not irrational (ie. rational)."

Prove the following statement.

SCORE: ___ / 4 POINTS

DO NOT USE ANY THEOREMS OR EXERCISES FROM THE BOOK AS JUSTIFICATION.

The difference of any irrational number minus any rational number is irrational.

Proof by contradiction:

Assume the negation of the statement, that there exists an irrational number x, and a rational number y, such that $x - y$ is rational.

By definition of rational, $y = \frac{a}{b}$ and $x - y = \frac{c}{d}$ for some integers a, b, c, d such that $b \neq 0$ and $d \neq 0$.

So $x = y + (x - y) = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

where $ad + bc$ and bd are integers by the closure of integers under multiplication and addition,

and $bd \neq 0$ by the zero product property.

Therefore, by definition of rational, x is rational, which contradicts the assumption that x is irrational.

By contradiction, the negation is false, and the original statement is true.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Pop Quiz 4 Version S
Thu Oct 27, 2011

SCORE: ___ / 6 POINTS

Consider the true statement "If $x \bmod 6 = 4$, then $(-x) \bmod 6 = 2$". **SCORE: ___ / 1 POINT**
Find the **FIRST** error in the following incorrect "proof", and explain in **10 WORDS OR FEWER** what the error is.

"Proof": Let x be a particular but arbitrary integer such that $x \bmod 6 = 4$.
By the definition of mod, $x = 6k + 4$ for some integer k.
$\boxed{\text{If } (-x) \bmod 6 = 2}$, then by the definition of mod, $-x = 6q + 2$ for some integer q.
But $-x = -(6k+4) = -6k - 4$.
So, $6q + 2 = -6k - 4$.
So, $6q + 6k = -6$.
So, $q + k = -1$.
$q + k$ is an integer by the closure of integers under addition, and -1 is an integer, so $q + k$ could equal -1.
So, $(-x) \bmod 6 = 2$.

The writer assumed what was to be proved was already true.

Consider the true statement "The square root of an irrational number is irrational". **SCORE: ___ / 1 POINT**
Write **ONLY THE FIRST SENTENCE** of a proof by contradiction.

"Assume there is an irrational number whose square root is not irrational (ie. rational)."

Prove the following statement. **SCORE: ___ / 4 POINTS**
DO NOT USE ANY THEOREMS OR EXERCISES FROM THE BOOK AS JUSTIFICATION.

The difference of any irrational number minus any rational number is irrational.

Proof by contradiction:

Assume the negation of the statement, that there exists an irrational number x, and a rational number y, such that $x - y$ is rational.

By definition of rational, $y = \frac{a}{b}$ and $x - y = \frac{c}{d}$ for some integers a, b, c, d such that $b \neq 0$ and $d \neq 0$.

So $x = y + (x - y) = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
where $ad + bc$ and bd are integers by the closure of integers under multiplication and addition,
and $bd \neq 0$ by the zero product property.
Therefore, by definition of rational, x is rational, which contradicts the assumption that x is irrational.
By contradiction, the negation is false, and the original statement is true.