Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ___________**
Pop Quiz 5 Version S
Wed Nov 2, 2011

SCORE: ___ / 6 POINTS

Prove the following statement by mathematical induction. **SCORE: ___ / 6 POINTS**

For all integers $n \ge 0$, $$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2n+1)\times(2n+3)}=\frac{n+1}{2n+3}$$

BASIS STEP:

If $n = 0$, $\frac{1}{1\times 3}=\frac{1}{3}=\frac{0+1}{2(0)+3}$

INDUCTIVE STEP:

Assume that $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+1)\times(2k+3)}=\frac{k+1}{2k+3}$

for some particular but arbitrary integer k, where $k \ge 0$.

NEED TO SHOW:

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2(k+1)+1)\times(2(k+1)+3)}=\frac{(k+1)+1}{2(k+1)+3}$$

ie. $$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+3)\times(2k+5)}=\frac{k+2}{2k+5}$$

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+3)\times(2k+5)}$$
$$=\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+1)\times(2k+3)}+\frac{1}{(2k+3)\times(2k+5)}$$
$$=\frac{k+1}{2k+3}+\frac{1}{(2k+3)(2k+5)}$$
$$=\frac{(k+1)(2k+5)+1}{(2k+3)(2k+5)}$$
$$=\frac{2k^2+7k+6}{(2k+3)(2k+5)}$$
$$=\frac{(2k+3)(k+2)}{(2k+3)(2k+5)}$$
$$=\frac{k+2}{2k+5}$$

By mathematical induction, $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2n+1)\times(2n+3)}=\frac{n+1}{2n+3}$ for all integers $n \ge 0$.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Pop Quiz 5 Version C
Wed Nov 2, 2011

SCORE: ___ / 6 POINTS

Prove the following statement by mathematical induction. **SCORE: ___ / 6 POINTS**

For all integers $n \geq 0$, $$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n+1)\times(3n+4)}=\frac{n+1}{3n+4}$$

BASIS STEP:

If $n = 0$, $\frac{1}{1\times 4}=\frac{1}{4}=\frac{0+1}{3(0)+4}$

INDUCTIVE STEP:

Assume that $$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k+1)\times(3k+4)}=\frac{k+1}{3k+4}$$

for some particular but arbitrary integer k, where $k \geq 0$.

NEED TO SHOW:

$$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3(k+1)+1)\times(3(k+1)+4)}=\frac{(k+1)+1}{3(k+1)+4}$$

ie. $$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k+4)\times(3k+7)}=\frac{k+2}{3k+7}$$

$$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k+4)\times(3k+7)}$$
$$=\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k+1)\times(3k+4)}+\frac{1}{(3k+4)\times(3k+7)}$$
$$=\frac{k+1}{3k+4}+\frac{1}{(3k+4)(3k+7)}$$
$$=\frac{(k+1)(3k+7)+1}{(3k+4)(3k+7)}$$
$$=\frac{3k^2+10k+8}{(3k+4)(3k+7)}$$
$$=\frac{(3k+4)(k+2)}{(3k+4)(3k+7)}$$
$$=\frac{k+2}{3k+7}$$

By mathematical induction, $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n+1)\times(3n+4)}=\frac{n+1}{3n+4}$ for all integers $n \geq 0$.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Pop Quiz 5 Version A
Wed Nov 2, 2011

SCORE: ___ / 6 POINTS

Prove the following statement by mathematical induction. **SCORE: ___ / 6 POINTS**

For all integers $n \geq 0$, $$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4n+1)\times(4n+5)}=\frac{n+1}{4n+5}$$

BASIS STEP:

If $n=0$, $\frac{1}{1\times 5}=\frac{1}{5}=\frac{0+1}{4(0)+5}$

INDUCTIVE STEP:

Assume that $$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4k+1)\times(4k+5)}=\frac{k+1}{4k+5}$$
for some particular but arbitrary integer k, where $k \geq 0$.

NEED TO SHOW:
$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4(k+1)+1)\times(4(k+1)+5)}=\frac{(k+1)+1}{4(k+1)+5}$$
ie. $$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4k+5)\times(4k+9)}=\frac{k+2}{4k+9}$$

$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4k+5)\times(4k+9)}$$
$$=\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4k+1)\times(4k+5)}+\frac{1}{(4k+5)\times(4k+9)}$$
$$=\frac{k+1}{4k+5}+\frac{1}{(4k+5)(4k+9)}$$
$$=\frac{(k+1)(4k+9)+1}{(4k+5)(4k+9)}$$
$$=\frac{4k^2+13k+10}{(4k+5)(4k+9)}$$
$$=\frac{(4k+5)(k+2)}{(4k+5)(4k+9)}$$
$$=\frac{k+2}{4k+9}$$

By mathematical induction, $\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(4n+1)\times(4n+5)}=\frac{n+1}{4n+5}$ for all integers $n \geq 0$.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ___________**
Pop Quiz 5 Version R
Wed Nov 2, 2011

SCORE: ___ / 6 POINTS

Prove the following statement by mathematical induction. **SCORE: ___ / 6 POINTS**

For all integers $n \ge 0$, $$\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n+1)\times(5n+6)}=\frac{n+1}{5n+6}$$

BASIS STEP:

If $n=0$, $\frac{1}{1\times 6}=\frac{1}{6}=\frac{0+1}{5(0)+6}$

INDUCTIVE STEP:

Assume that $\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5k+1)\times(5k+6)}=\frac{k+1}{5k+6}$

for some particular but arbitrary integer k, where $k \ge 0$.

NEED TO SHOW:

$$\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5(k+1)+1)\times(5(k+1)+6)}=\frac{(k+1)+1}{5(k+1)+6}$$

ie. $$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(5k+6)\times(5k+11)}=\frac{k+2}{5k+11}$$

$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(5k+6)\times(5k+11)}$$
$$=\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\cdots+\frac{1}{(5k+1)\times(5k+6)}+\frac{1}{(5k+6)\times(5k+11)}$$
$$=\frac{k+1}{5k+6}+\frac{1}{(5k+6)(5k+11)}$$
$$=\frac{(k+1)(5k+11)+1}{(5k+6)(5k+11)}$$
$$=\frac{5k^2+16k+12}{(5k+6)(5k+11)}$$
$$=\frac{(5k+6)(k+2)}{(5k+6)(5k+11)}$$
$$=\frac{k+2}{5k+11}$$

By mathematical induction, $\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n+1)\times(5n+6)}=\frac{n+1}{5n+6}$ for all integers $n \ge 0$.