

SCORE: ____ / 24 POINTS

Let $A = \{1, 4, 5, 7\}$ and $B = \{2, 3, 6, 8, 10\}$.

SCORE: ____ / 2 POINTS

Let $H = \{(x, 10), (5, y), (7, 2), (1, 3), (5, 8)\} \subseteq A \times B$.

[a] If H is a function, what is the value of x ?

4

[b] If H is a function, what is the value of y ?

8

Consider the statement form $(\sim p \rightarrow q) \leftrightarrow p$.

SCORE: ____ / 4 POINTS

[a] Construct the truth table.

p	q	$\sim p$	$\sim p \rightarrow q$	$(\sim p \rightarrow q) \leftrightarrow p$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

[b] Circle the statement form below that is logically equivalent to the statement form above.

p	q	$\sim p$	$\sim q$	t	c
$p \wedge q$	$p \vee q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$p \oplus q$

Write the negation of the following statement.

SCORE: ____ / 2 POINTS

“My car is a mess, but the windows are clean.”

$\sim(\text{“My car is a mess”} \wedge \text{“the windows are clean”})$

$\sim\text{“My car is a mess”} \vee \sim\text{“the windows are clean”}$

“My car is not a mess, or the windows are not clean”

The official motto of New Hampshire is “Live free or die”, which can be translated as “I live free or I die”.

SCORE: ____ / 2 POINTS

Write a logically equivalent statement using “if/then”.

$\sim\text{“I do not live free”} \vee \text{“I die”}$

OR

$\sim\text{“I do not die”} \vee \text{“I live free”}$

“If I do not live free, then I die”

“If I do not die, then I live free”

Consider the statement “ $2 - x < 3$ is necessary for $x > -1$ ”.

SCORE: ____ / 5 POINTS

[a] Write a logically equivalent statement using “if/then”, “ $2 - x < 3$ ” and “ $x > -1$ ”.

“If $x > -1$, then $2 - x < 3$ ”

[b] Write the inverse of your answer in [a].

“If $x \leq -1$, then $2 - x \geq 3$ ”

[c] Write the negation of the original statement **WITHOUT** using any of the following: “necessary”, “sufficient”, “if”, “not”.

“ $x > -1$ and $2 - x \geq 3$ ”

The following argument uses Modus Tollens. Fill in the missing statements.

SCORE: ____ / 2 POINTS

If my age is divisible by 4, then it is not prime.

My age is prime.

Therefore, **my age is not divisible by 4.**

Write the logical form of the following argument, then use **truth tables** to determine if it is valid.

SCORE: ____ / 7 POINTS

Mark the critical rows clearly, and STATE CLEARLY WHETHER THE ARGUMENT IS VALID OR INVALID.

If Petra pounces, then Quentin does not quiver.

Petra pounces or Rula reacts.

Therefore, if Quentin quivers, then Rula reacts.

$p \rightarrow \sim q$

$p \vee r$

$\therefore q \rightarrow r$

p	q	r	$\sim q$	$p \rightarrow \sim q$	$p \vee r$	$q \rightarrow r$
T	T	T	F	F	T	T
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	T

The argument is valid.

SCORE: ____ / 24 POINTS

Let $A = \{1, 4, 5, 7\}$ and $B = \{2, 3, 6, 8, 10\}$.

SCORE: ____ / 2 POINTS

Let $H = \{(x, 10), (7, y), (4, 3), (5, 8), (7, 6)\} \subseteq A \times B$.

[a] If H is a function, what is the value of x ?

1

[b] If H is a function, what is the value of y ?

6

Consider the statement form $(\sim p \rightarrow q) \leftrightarrow q$.

SCORE: ____ / 4 POINTS

[a] Construct the truth table.

p	q	$\sim p$	$\sim p \rightarrow q$	$(\sim p \rightarrow q) \leftrightarrow q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

[b] Circle the statement form below that is logically equivalent to the statement form above.

p	q	$\sim p$	$\sim q$	t	c
$p \wedge q$	$p \vee q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$p \oplus q$

Write the negation of the following statement.

SCORE: ____ / 2 POINTS

“My car is clean, but the windows are broken.”

$\sim(\text{“My car is clean”} \wedge \text{“the windows are broken”})$

$\sim\text{“My car is clean”} \vee \sim\text{“the windows are broken”}$

“My car is not clean, or the windows are not broken”

The official motto of New Hampshire is “Live free or die”, which can be translated as “I live free or I die”.

SCORE: ____ / 2 POINTS

Write a logically equivalent statement using “if/then”.

$\sim\text{“I do not live free”} \vee \text{“I die”}$

OR

$\sim\text{“I do not die”} \vee \text{“I live free”}$

“If I do not live free, then I die”

“If I do not die, then I live free”

Consider the statement “ $x > -1$ is necessary for $2 - x < 3$ ”.

SCORE: ____ / 5 POINTS

[a] Write a logically equivalent statement using “if/then”, “ $x > -1$ ” and “ $2 - x < 3$ ”.

“If $2 - x < 3$, then $x > -1$ ”

[b] Write the converse of your answer in [a].

“If $x > -1$, then $2 - x < 3$ ”

[c] Write the negation of the original statement **WITHOUT** using any of the following: “necessary”, “sufficient”, “if”, “not”.

“ $2 - x < 3$ and $x \leq -1$ ”

The following argument uses Modus Tollens. Fill in the missing statements.

SCORE: ____ / 2 POINTS

If my age is prime, then it is not divisible by 4.

My age is divisible by 4.

Therefore, **my age is not prime.**

Write the logical form of the following argument, then use **truth tables** to determine if it is valid.

SCORE: ____ / 7 POINTS

Mark the critical rows clearly, and STATE CLEARLY WHETHER THE ARGUMENT IS VALID OR INVALID.

If Petra pounces, then Quentin quivers

Petra pounces or Rula reacts.

Therefore, if Quentin quivers, then Rula does not react.

$p \rightarrow q$

$p \vee r$

$\therefore q \rightarrow \sim r$

p	q	r	$\sim r$	$p \rightarrow q$	$p \vee r$	$q \rightarrow \sim r$
T	T	T	F	T	T	F
T	T	F	T	T	T	T
T	F	T	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	F	T	F	T	T	T
F	F	F	T	T	F	T

The argument is invalid.

SCORE: ____ / 24 POINTS

Let $A = \{1, 4, 5, 7\}$ and $B = \{2, 3, 6, 8, 10\}$.

SCORE: ____ / 2 POINTS

Let $H = \{(x, 3), (1, y), (5, 2), (4, 8), (1, 10)\} \subseteq A \times B$.

[a] If H is a function, what is the value of x ?

7

[b] If H is a function, what is the value of y ?

10

Consider the statement form $\sim p \rightarrow (p \leftrightarrow q)$.

SCORE: ____ / 4 POINTS

[a] Construct the truth table.

p	q	$\sim p$	$p \leftrightarrow q$	$\sim p \rightarrow (p \leftrightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T

[b] Circle the statement form below that is logically equivalent to the statement form above.

p q $\sim p$ $\sim q$ t c
 $p \wedge q$ $p \vee q$ $p \rightarrow q$ **$q \rightarrow p$** $p \leftrightarrow q$ $p \oplus q$

Write the negation of the following statement.

SCORE: ____ / 2 POINTS

“My car is broken, but the windows are new.”

$\sim(\text{“My car is broken”} \wedge \text{“the windows are new”})$

$\sim\text{“My car is broken”} \vee \sim\text{“the windows are new”}$

“My car is not broken, or the windows are not new”

The official motto of New Hampshire is “Live free or die”, which can be translated as “I live free or I die”.

SCORE: ____ / 2 POINTS

Write a logically equivalent statement using “if/then”.

$\sim\text{“I do not live free”} \vee \text{“I die”}$

OR

$\sim\text{“I do not die”} \vee \text{“I live free”}$

“If I do not live free, then I die”

“If I do not die, then I live free”

Consider the statement “ $x > -2$ is necessary for $2 - x < 4$ ”.

SCORE: ____ / 5 POINTS

[a] Write a logically equivalent statement using “if/then”, “ $x > -2$ ” and “ $2 - x < 4$ ”.

“If $2 - x < 4$, then $x > -2$ ”

[b] Write the contrapositive of your answer in [a].

“If $x \leq -2$, then $2 - x \geq 4$ ”

[c] Write the negation of the original statement **WITHOUT** using any of the following: “necessary”, “sufficient”, “if”, “not”.

“ $2 - x < 4$ and $x \leq -2$ ”

The following argument uses Modus Ponens. Fill in the missing statements.

SCORE: ____ / 2 POINTS

If my age is prime, then it is not divisible by 4.

My age is prime.

Therefore, **my age is not divisible by 4.**

Write the logical form of the following argument, then use **truth tables** to determine if it is valid.

SCORE: ____ / 7 POINTS

Mark the critical rows clearly, and STATE CLEARLY WHETHER THE ARGUMENT IS VALID OR INVALID.

If Petra pounces, then Quentin quivers.

Petra pounces or Rula reacts.

Therefore, if Quentin does not quiver, then Rula reacts.

$p \rightarrow q$

$p \vee r$

$\therefore \sim q \rightarrow r$

p	q	r	$\sim q$	$p \rightarrow q$	$p \vee r$	$\sim q \rightarrow r$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

The argument is valid.

SCORE: ____ / 24 POINTS

Let $A = \{1, 4, 5, 7\}$ and $B = \{2, 3, 6, 8, 10\}$.

SCORE: ____ / 2 POINTS

Let $H = \{(x, 10), (4, y), (7, 2), (1, 8), (4, 3)\} \subseteq A \times B$.

[a] If H is a function, what is the value of x ?

5

[b] If H is a function, what is the value of y ?

3

Consider the statement form $\sim q \rightarrow (p \leftrightarrow q)$.

SCORE: ____ / 4 POINTS

[a] Construct the truth table.

p	q	$\sim q$	$p \leftrightarrow q$	$\sim q \rightarrow (p \leftrightarrow q)$
T	T	F	T	T
T	F	T	F	F
F	T	F	F	T
F	F	T	T	T

[b] Circle the statement form below that is logically equivalent to the statement form above.

p	q	$\sim p$	$\sim q$	t	c
$p \wedge q$	$p \vee q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$p \oplus q$

Write the negation of the following statement.

SCORE: ____ / 2 POINTS

“My car is new, but the windows are dirty.”

$\sim(\text{“My car is new”} \wedge \text{“the windows are dirty”})$

$\sim\text{“My car is new”} \vee \sim\text{“the windows are dirty”}$

“My car is not new, or the windows are not dirty”

The official motto of New Hampshire is “Live free or die”, which can be translated as “I live free or I die”.

SCORE: ____ / 2 POINTS

Write a logically equivalent statement using “if/then”.

$\sim\text{“I do not live free”} \vee \text{“I die”}$

OR

$\sim\text{“I do not die”} \vee \text{“I live free”}$

“If I do not live free, then I die”

“If I do not die, then I live free”

Consider the statement “ $2 - x < 4$ is necessary for $x > -2$ ”.

SCORE: ____ / 5 POINTS

[a] Write a logically equivalent statement using “if/then”, “ $2 - x < 4$ ” and “ $x > -2$ ”.

“If $x > -2$, then $2 - x < 4$ ”

[b] Write the converse of your answer in [a].

“If $2 - x < 4$, then $x > -2$ ”

[c] Write the negation of the original statement **WITHOUT** using any of the following: “necessary”, “sufficient”, “if”, “not”.

“ $x > -2$ and $2 - x \geq 4$ ”

The following argument uses Modus Ponens. Fill in the missing statements.

SCORE: ____ / 2 POINTS

If my age is divisible by 4, then it is not prime.

My age is divisible by 4.

Therefore, **my age is not prime.**

Write the logical form of the following argument, then use **truth tables** to determine if it is valid.

SCORE: ____ / 7 POINTS

Mark the critical rows clearly, and STATE CLEARLY WHETHER THE ARGUMENT IS VALID OR INVALID.

If Petra pounces, then Quentin does not quiver.

Petra pounces or Rula reacts.

Therefore, if Rula reacts, then Quentin quivers.

$p \rightarrow \sim q$

$p \vee r$

$\therefore r \rightarrow q$

p	q	r	$\sim q$	$p \rightarrow \sim q$	$p \vee r$	$r \rightarrow q$
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	T	T	T	F
F	F	F	T	T	F	T

The argument is invalid.