Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Quiz 3 Version F
Fri Oct 21, 2011

SCORE: ___ / 24 POINTS

Fill in the blanks. Show **very briefly** why your answer is correct. **SCORE: ___ / 4 POINTS**

[a] $-42 \bmod 5 = 3$ $-42 = 5 \times (-9) + 3$

[b] $-42 \; div \; 5 = -9$

One of the statements below is true, and the other is false. **SCORE: ___ / 4 POINTS**
Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

[a] $\exists n \in Z^+ : 2^n + 3$ is composite

TRUE: $n = 5 \rightarrow 2^n + 3 = 35 = 5 \times 7$

[b] The set of non-zero rational numbers is closed under addition.

FALSE: $\frac{1}{2} + \frac{-1}{2} = 0$

Prove the following statement. **SCORE: ___ / 5 POINTS**

For all integers a and b,
if $a \bmod 5 = 2$ and $b \bmod 5 = 3$, then $ab \bmod 5 = 1$.

PROOF:

Let a and b be particular but arbitrary integers such that $a \bmod 5 = 2$ and $b \bmod 5 = 3$.
By the definition of mod, $a = 5m + 2$ and $b = 5n + 3$ for some integers m and n.
So, $ab = (5m+2)(5n+3) = 25mn + 15m + 10n + 6 = 5(5mn + 3m + 2n + 1) + 1$
where $5mn + 3m + 2n + 1$ is an integer by the closure of integers under addition and multiplication, and $0 \le 1 < 5$.
So, by the definition of mod, $ab \bmod 5 = 1$.

Prove that the set of non-zero rational numbers is closed under division. **SCORE: ___ / 6 POINTS**

PROOF:

Let a and b be particular but arbitrary non-zero rational numbers.

By the definition of rational, $a = \dfrac{p}{q}$ and $b = \dfrac{r}{s}$ for some integers p, q, r and s where $q \neq 0$ and $s \neq 0$.

Since $a \neq 0$ and $b \neq 0$, therefore $p \neq 0$ and $r \neq 0$.

$\dfrac{a}{b} = \dfrac{ps}{qr}$ where ps and qr are integers by the closure of integers under multiplication,

and $ps \neq 0$ and $qr \neq 0$ by zero product property.

Therefore, by the definition of rational, $\dfrac{a}{b}$ is a non-zero rational number.

Prove the following statement. **SCORE: ___ / 5 POINTS**

The difference between the squares of any two consecutive odd integers is divisible by 8.
(NOTE: Two consecutive odd integers are two odd integers whose difference is 2.)

PROOF:

Let x and $x+2$ be two particular but arbitrary consecutive odd integers.
By definition of odd, $x = 2k+1$ for some integer k.
Therefore, $(x+2)^2 - x^2 = 4x+4 = 4(2k+1)+4 = 8k+8 = 8(k+1)$
where $k+1$ is an integer under the closure of integers under addition.
So, by the definition of divisibility, the difference between the squares of two consecutive odd integers is divisible by 8.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Quiz 3 Version L
Fri Oct 21, 2011

SCORE: ___ / 24 POINTS

Fill in the blanks. Show **very briefly** why your answer is correct. **SCORE: ___ / 4 POINTS**

[a] $-34 \bmod 5 = 1$ $\qquad -34 = 5 \times (-7) + 1$

[b] $-34 \, div \, 5 = -7$

One of the statements below is true, and the other is false. **SCORE: ___ / 4 POINTS**
Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

[a] $\exists n \in Z^+ : 2^n + 9$ is composite

TRUE: $n = 4 \rightarrow 2^n + 9 = 25 = 5 \times 5$

[b] The set of non-zero rational numbers is closed under addition.

FALSE: $\frac{1}{2} + \frac{-1}{2} = 0$

Prove the following statement. **SCORE: ___ / 5 POINTS**

The difference between the squares of any two consecutive odd integers is divisible by 8.
(NOTE: Two consecutive odd integers are two odd integers whose difference is 2.)

PROOF:

Let x and $x+2$ be two particular but arbitrary consecutive odd integers.
By definition of odd, $x = 2k+1$ for some integer k.
Therefore, $(x+2)^2 - x^2 = 4x + 4 = 4(2k+1) + 4 = 8k + 8 = 8(k+1)$
where $k+1$ is an integer under the closure of integers under addition.
So, by the definition of divisibility, the difference between the squares of two consecutive odd integers is divisible by 8.

Prove the following statement. **SCORE: ___ / 5 POINTS**

For all integers a and b,
if $a \bmod 5 = 3$ and $b \bmod 5 = 4$, then $ab \bmod 5 = 2$.

PROOF:

Let a and b be particular but arbitrary integers such that $a \bmod 5 = 3$ and $b \bmod 5 = 4$.
By the definition of mod, $a = 5m + 3$ and $b = 5n + 4$ for some integers m and n.
So, $ab = (5m+3)(5n+4) = 25mn + 20m + 15n + 12 = 5(5mn + 4m + 3n + 2) + 2$
where $5mn + 4m + 3n + 2$ is an integer by the closure of integers under addition and multiplication, and $0 \le 2 < 5$.
So, by the definition of mod, $ab \bmod 5 = 2$.

Prove that the set of non-zero rational numbers is closed under division. **SCORE: ___ / 6 POINTS**

PROOF:

Let a and b be particular but arbitrary non-zero rational numbers.

By the definition of rational, $a = \frac{p}{q}$ and $b = \frac{r}{s}$ for some integers p, q, r and s where $q \ne 0$ and $s \ne 0$.

Since $a \ne 0$ and $b \ne 0$, therefore $p \ne 0$ and $r \ne 0$.

$\frac{a}{b} = \frac{ps}{qr}$ where ps and qr are integers by the closure of integers under multiplication,

and $ps \ne 0$ and $qr \ne 0$ by zero product property.

Therefore, by the definition of rational, $\frac{a}{b}$ is a non-zero rational number.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Quiz 3 Version A
Fri Oct 21, 2011

SCORE: ___ / 24 POINTS

Fill in the blanks. Show **<u>very briefly</u>** why your answer is correct. **SCORE: ___ / 4 POINTS**

[a] $-19 \bmod 6 = 5$ $\qquad -19 = 6 \times (-4) + 5$

[b] $-19\, div\, 6 = -4$

One of the statements below is true, and the other is false. **SCORE: ___ / 4 POINTS**
Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

[a] $\exists n \in Z^+ : n^2 + n + 1$ is composite

TRUE: $n = 4 \rightarrow n^2 + n + 1 = 21 = 3 \times 7$

[b] The set of non-zero rational numbers is closed under addition.

FALSE: $\frac{1}{2} + \frac{-1}{2} = 0$

Prove that the set of non-zero rational numbers is closed under division. **SCORE: ___ / 6 POINTS**

PROOF:

Let a and b be particular but arbitrary non-zero rational numbers.

By the definition of rational, $a = \frac{p}{q}$ and $b = \frac{r}{s}$ for some integers p, q, r and s where $q \neq 0$ and $s \neq 0$.

Since $a \neq 0$ and $b \neq 0$, therefore $p \neq 0$ and $r \neq 0$.

$\frac{a}{b} = \frac{ps}{qr}$ where ps and qr are integers by the closure of integers under multiplication,

and $ps \neq 0$ and $qr \neq 0$ by zero product property.

Therefore, by the definition of rational, $\frac{a}{b}$ is a non-zero rational number.

Prove the following statement. **SCORE: ___ / 5 POINTS**

The difference between the squares of any two consecutive odd integers is divisible by 8.
(NOTE: Two consecutive odd integers are two odd integers whose difference is 2.)

PROOF:

Let x and $x+2$ be two particular but arbitrary consecutive odd integers.
By definition of odd, $x = 2k+1$ for some integer k.
Therefore, $(x+2)^2 - x^2 = 4x+4 = 4(2k+1)+4 = 8k+8 = 8(k+1)$
where $k+1$ is an integer under the closure of integers under addition.
So, by the definition of divisibility, the difference between the squares of two consecutive odd integers is divisible by 8.

Prove the following statement. **SCORE: ___ / 5 POINTS**

For all integers a and b,
if $a \bmod 7 = 3$ and $b \bmod 7 = 4$, then $ab \bmod 7 = 5$.

PROOF:

Let a and b be particular but arbitrary integers such that $a \bmod 7 = 3$ and $b \bmod 7 = 4$.
By the definition of mod, $a = 7m+3$ and $b = 7n+4$ for some integers m and n.
So, $ab = (7m+3)(7n+4) = 49mn + 28m + 21n + 12 = 7(7mn + 4m + 3n + 1) + 5$
where $7mn + 4m + 3n + 1$ is an integer by the closure of integers under addition and multiplication, and $0 \le 5 < 7$.
So, by the definition of mod, $ab \bmod 7 = 5$.

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**
Quiz 3 Version G
Fri Oct 21, 2011

SCORE: ___ / 24 POINTS

Fill in the blanks. Show **very briefly** why your answer is correct. **SCORE: ___ / 4 POINTS**

[a] $-26 \bmod 6 = 4$ $-26 = 6 \times (-5) + 4$

[b] $-26 \, div \, 6 = -5$

One of the statements below is true, and the other is false. **SCORE: ___ / 4 POINTS**
Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

[a] $\exists n \in Z^+ : n^3 + n + 1$ is composite

TRUE: $n = 4 \rightarrow n^3 + n + 1 = 69 = 3 \times 23$

[b] The set of non-zero rational numbers is closed under addition.

FALSE: $\frac{1}{2} + \frac{-1}{2} = 0$

Prove the following statement. **SCORE: ___ / 5 POINTS**

The difference between the squares of any two consecutive odd integers is divisible by 8.
(NOTE: Two consecutive odd integers are two odd integers whose difference is 2.)

PROOF:

Let x and $x+2$ be two particular but arbitrary consecutive odd integers.
By definition of odd, $x = 2k+1$ for some integer k.
Therefore, $(x+2)^2 - x^2 = 4x + 4 = 4(2k+1) + 4 = 8k + 8 = 8(k+1)$
where $k+1$ is an integer under the closure of integers under addition.
So, by the definition of divisibility, the difference between the squares of two consecutive odd integers is divisible by 8.

Prove that the set of non-zero rational numbers is closed under division. **SCORE: ___ / 6 POINTS**

PROOF:

Let a and b be particular but arbitrary non-zero rational numbers.

By the definition of rational, $a = \frac{p}{q}$ and $b = \frac{r}{s}$ for some integers p, q, r and s where $q \neq 0$ and $s \neq 0$.

Since $a \neq 0$ and $b \neq 0$, therefore $p \neq 0$ and $r \neq 0$.

$\frac{a}{b} = \frac{ps}{qr}$ where ps and qr are integers by the closure of integers under multiplication,

and $ps \neq 0$ and $qr \neq 0$ by zero product property.

Therefore, by the definition of rational, $\frac{a}{b}$ is a non-zero rational number.

Prove the following statement. **SCORE: ___ / 5 POINTS**

For all integers a and b,
if $a \bmod 7 = 2$ and $b \bmod 7 = 5$, then $ab \bmod 7 = 3$.

PROOF:

Let a and b be particular but arbitrary integers such that $a \bmod 7 = 2$ and $b \bmod 7 = 5$.
By the definition of mod, $a = 5m + 2$ and $b = 5n + 3$ for some integers m and n.
So, $ab = (7m+2)(7n+5) = 49mn + 35m + 14n + 10 = 7(7mn + 5m + 2n + 1) + 3$
where $7mn + 5m + 2n + 1$ is an integer by the closure of integers under addition and multiplication, and $0 \leq 3 < 7$.
So, by the definition of mod, $ab \bmod 7 = 3$.