Math 22 (9:30am – 10:20am)
Quiz 4 Version T
Fri Oct 21, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 24 POINTS

Prove the following statement. **SCORE: ___ / 8 POINTS**

The sum of the squares of any two consecutive integers has the form $4k+1$.

Do NOT use the property that consecutive integers have opposite parity, nor the table of properties regarding the arithmetic of even and odd numbers, unless you prove them here.

Let x be an integer.
$x^2+(x+1)^2 = 2x^2+2x+1 = 2x(x+1)+1$
By parity property, x is either even or odd.
Case 1: If x is even,
then by definition of even, $x=2m$ for some $m \in Z$.
$2x(x+1)+1 = 2(2m)(2m+1)+1 = 4m(2m+1)+1$
where $m(2m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
Case 2: If x is odd,
then by definition of odd, $x=2m+1$ for some $m \in Z$.
$2x(x+1)+1 = 2(2m+1)(2m+2)+1 = 4(2m+1)(m+1)+1$
where $(2m+1)(m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
So, the sum of the squares of two consecutive integers, always has the form $4k+1$ for some $k \in Z$.

Based on a question from your homework: If you wanted to show that 173 is prime, what is the largest number you need to try to divide 173 by ? Show **very briefly** how you got your answer. **SCORE: ___ / 3 POINTS**

13 is the largest prime less than $\sqrt{173}$

Prove that there is no least positive rational number. **SCORE: ___ / 8 POINTS**

Assume that there is a least positive rational number, which we call x.

By definition of rational, $x = \frac{a}{b}$ for some $a, b \in Z$ where $b \neq 0$.

Since $x > 0$, therefore $\frac{1}{2}x > 0$ [by multiplying both sides of the inequality by $\frac{1}{2}$].

Since $\frac{1}{2} < 1$ and $x > 0$, therefore $\frac{1}{2}x < x$ [by multiplying both sides of the first inequality by x].

In addition, $\frac{1}{2}x = \frac{a}{2b}$ where $b \in Z$ by the closure of Z under multiplication, and $2b \neq 0$ by the zero product property.

So, by definition of rational, $\frac{1}{2}x$ is a rational number, which is also positive and less than x.

But this contradicts the assumption that x was the least positive rational number.
So, the assumption is wrong, and there is no least positive rational number.

Consider the true statement "If x is odd, then x^2 is odd". **SCORE: ___ / 2 POINTS**
Find the **FIRST** error in the following incorrect "proof" by contraposition, and explain in **12 WORDS OR FEWER** what the error is.

"Proof": Suppose that x is not odd.
By the parity property, x is even.
By the definition of even, $x = 2k$ for some integer k.
So, $x^2 = (2k)^2 = 4k^2 = 2(2k^2)$.
$2k^2$ is an integer by the closure of integers under multiplication.
So, by the definition of even, x^2 is even.
By the parity property, x^2 is not odd.
So, the statement is true.

The first sentence is not the negation of the conclusion.

Exactly one of the following statements is <u>**false**</u>. Find the statement, and find a counterexample. **SCORE: ___ / 3 POINTS**
You do not have to prove why your counterexample is correct, but it must be correct.

[I] The quotient of an irrational number divided by a non-zero rational number must be irrational.
[II] The reciprocal of an irrational number must be irrational.
[III] The sum of two positive irrational numbers must be irrational.

[III] $10 - \sqrt{2}$ and $10 + \sqrt{2}$ are both positive and irrational, but $10 - \sqrt{2} + 10 + \sqrt{2} = 20$ is not

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SCORE: ___ / 24 POINTS

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The sum of the squares of any two consecutive integers has the form $4k+1$.

Do NOT use the property that consecutive integers have opposite parity, nor the table of properties regarding the arithmetic of even and odd numbers, unless you prove them here.

Let x be an integer.
$x^2+(x+1)^2=2x^2+2x+1=2x(x+1)+1$
By parity property, x is either even or odd.
Case 1: If x is even,
then by definition of even, $x=2m$ for some $m \in Z$.
$2x(x+1)+1=2(2m)(2m+1)+1=4m(2m+1)+1$
where $m(2m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
Case 2: If x is odd,
then by definition of odd, $x=2m+1$ for some $m \in Z$.
$2x(x+1)+1=2(2m+1)(2m+2)+1=4(2m+1)(m+1)+1$
where $(2m+1)(m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
So, the sum of the squares of two consecutive integers, always has the form $4k+1$ for some $k \in Z$.

Based on a question from your homework: If you wanted to show that 173 is prime, what is the largest number you need to try to divide 173 by ? Show **very briefly** how you got your answer. **SCORE: ___ / 3 POINTS**

13 is the largest prime less than $\sqrt{173}$

Prove that there is no least positive rational number. **SCORE: ___ / 8 POINTS**

Assume that there is a least positive rational number, which we call x.

By definition of rational, $x = \frac{a}{b}$ for some $a, b \in Z$ where $b \neq 0$.

Since $x > 0$, therefore $\frac{1}{2}x > 0$ [by multiplying both sides of the inequality by $\frac{1}{2}$].

Since $\frac{1}{2} < 1$ and $x > 0$, therefore $\frac{1}{2}x < x$ [by multiplying both sides of the first inequality by x].

In addition, $\frac{1}{2}x = \frac{a}{2b}$ where $b \in Z$ by the closure of Z under multiplication, and $2b \neq 0$ by the zero product property.

So, by definition of rational, $\frac{1}{2}x$ is a rational number, which is also positive and less than x.

But this contradicts the assumption that x was the least positive rational number.
So, the assumption is wrong, and there is no least positive rational number.

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"Proof": Suppose that x is not odd.
By the parity property, x is even.
By the definition of even, $x = 2k$ for some integer k.
So, $x^2 = (2k)^2 = 4k^2 = 2(2k^2)$.
$2k^2$ is an integer by the closure of integers under multiplication.
So, by the definition of even, x^2 is even.
By the parity property, x^2 is not odd.
So, the statement is true.

The first sentence is not the negation of the conclusion.

Exactly one of the following statements is **false**. Find the statement, and find a counterexample. **SCORE: ___ / 3 POINTS**
You do not have to prove why your counterexample is correct, but it must be correct.

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[II] The reciprocal of an irrational number must be irrational.
[III] The sum of two positive irrational numbers must be irrational.

[III] $10 - \sqrt{2}$ and $10 + \sqrt{2}$ are both positive and irrational, but $10 - \sqrt{2} + 10 + \sqrt{2} = 20$ is not

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SCORE: ___ / 24 POINTS

Consider the true statement "If x is odd, then x^2 is odd". **SCORE: ___ / 2 POINTS**
Find the **FIRST** error in the following incorrect "proof" by contraposition, and explain in **12 WORDS OR FEWER** what the error is.

"Proof": Suppose that x is not odd.
By the parity property, x is even.
By the definition of even, $x = 2k$ for some integer k.
So, $x^2 = (2k)^2 = 4k^2 = 2(2k^2)$.
$2k^2$ is an integer by the closure of integers under multiplication.
So, by the definition of even, x^2 is even.
By the parity property, x^2 is not odd.
So, the statement is true.

The first sentence is not the negation of the conclusion.

Prove that there is no least positive rational number. **SCORE: ___ / 8 POINTS**

Assume that there is a least positive rational number, which we call x.

By definition of rational, $x = \frac{a}{b}$ for some $a, b \in Z$ where $b \neq 0$.

Since $x > 0$, therefore $\frac{1}{2}x > 0$ [by multiplying both sides of the inequality by $\frac{1}{2}$].

Since $\frac{1}{2} < 1$ and $x > 0$, therefore $\frac{1}{2}x < x$ [by multiplying both sides of the first inequality by x].

In addition, $\frac{1}{2}x = \frac{a}{2b}$ where $b \in Z$ by the closure of Z under multiplication, and $2b \neq 0$ by the zero product property.

So, by definition of rational, $\frac{1}{2}x$ is a rational number, which is also positive and less than x.

But this contradicts the assumption that x was the least positive rational number.
So, the assumption is wrong, and there is no least positive rational number.

Exactly one of the following statements is **false**. Find the statement, and find a counterexample. You do not have to prove why your counterexample is correct, but it must be correct.

SCORE: ___ / 3 POINTS

[I] The quotient of an irrational number divided by a non-zero rational number must be irrational.
[II] The sum of two positive irrational numbers must be irrational.
[III] The reciprocal of an irrational number must be irrational.

[II] $10-\sqrt{2}$ and $10+\sqrt{2}$ are both positive and irrational, but $10-\sqrt{2}+10+\sqrt{2}=20$ is not

Based on a question from your homework: If you wanted to show that 163 is prime, what is the largest number you need to try to divide 163 by ? Show **very briefly** how you got your answer.

SCORE: ___ / 3 POINTS

11 is the largest prime less than $\sqrt{163}$

Prove the following statement.

SCORE: ___ / 8 POINTS

The sum of the squares of any two consecutive integers has the form $4k+1$.

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Let x be an integer.
$x^2+(x+1)^2=2x^2+2x+1=2x(x+1)+1$
By parity property, x is either even or odd.
Case 1: If x is even,
then by definition of even, $x=2m$ for some $m \in Z$.
$2x(x+1)+1=2(2m)(2m+1)+1=4m(2m+1)+1$
where $m(2m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
Case 2: If x is odd,
then by definition of odd, $x=2m+1$ for some $m \in Z$.
$2x(x+1)+1=2(2m+1)(2m+2)+1=4(2m+1)(m+1)+1$
where $(2m+1)(m+1) \in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k \in Z$.
So, the sum of the squares of two consecutive integers, always has the form $4k+1$ for some $k \in Z$.

Math 22 (9:30am – 10:20am)
Quiz 4 Version E
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NAME YOU ASKED TO BE CALLED IN CLASS: ____________

SCORE: ___ / 24 POINTS

Consider the true statement "If x is odd, then x^2 is odd". **SCORE: ___ / 2 POINTS**
Find the **FIRST** error in the following incorrect "proof" by contraposition, and explain in **12 WORDS OR FEWER** what the error is.

"Proof": Suppose that x is not odd.
By the parity property, x is even.
By the definition of even, $x = 2k$ for some integer k.
So, $x^2 = (2k)^2 = 4k^2 = 2(2k^2)$.
$2k^2$ is an integer by the closure of integers under multiplication.
So, by the definition of even, x^2 is even.
By the parity property, x^2 is not odd.
So, the statement is true.

The first sentence is not the negation of the conclusion.

Prove that there is no least positive rational number. **SCORE: ___ / 8 POINTS**

Assume that there is a least positive rational number, which we call x.

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In addition, $\frac{1}{2}x = \frac{a}{2b}$ where $b \in Z$ by the closure of Z under multiplication, and $2b \neq 0$ by the zero product property.

So, by definition of rational, $\frac{1}{2}x$ is a rational number, which is also positive and less than x.

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So, the sum of the squares has the form $4k+1$ for some $k\in Z$.
Case 2: If x is odd,
then by definition of odd, $x=2m+1$ for some $m\in Z$.
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where $(2m+1)(m+1)\in Z$ by closure of Z under multiplication and addition.
So, the sum of the squares has the form $4k+1$ for some $k\in Z$.
So, the sum of the squares of two consecutive integers, always has the form $4k+1$ for some $k\in Z$.