Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ____________**

Quiz 5 Version C

Fri Nov 4, 2011

SCORE: ___ / 24 POINTS

Evaluate $\binom{8}{5}$. **SCORE: ___ / 2 POINTS**

$$\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8\times 7\times 6\times 5!}{5!3!} = \frac{8\times 7\times 6}{3!} = \frac{8\times 7\times 6}{6} = 56$$

Prove that $5^n + 9 < 6^n$ for all integers $n \ge 2$ by mathematical induction. **SCORE: ___ / 9 POINTS**

BASIS STEP: $n = 2$: $5^2 + 9 = 34 < 36 = 6^2$

INDUCTIVE STEP: Assume $5^k + 9 < 6^k$ for some particular but arbitrary integer $k \ge 2$.

So, $5^k < 6^k - 9$

$$\begin{aligned} &5^{k+1} + 9 \\ &= 5\times 5^k + 9 \\ &< 5\times(6^k - 9) + 9 \\ &= 5\times 6^k - 45 + 9 \\ &= 5\times 6^k - 36 \\ &< 6\times 6^k - 36 \\ &< 6\times 6^k \\ &= 6^{k+1} \end{aligned}$$

OR

$$\begin{aligned} &6^{k+1} \\ &= 6\times 6^k \\ &> 6\times(5^k + 9) \\ &> 5\times(5^k + 9) \\ &= 5\times 5^k + 45 \\ &= 5^{k+1} + 45 \\ &> 5^{k+1} + 9 \end{aligned}$$

By mathematical induction, $5^n + 9 < 6^n$ for all integers $n \ge 2$.

Simplify $\dfrac{(n-2)!}{(n+1)!}$. **SCORE: ___ / 2 POINTS**

$$\frac{(n-2)!}{(n+1)!} = \frac{(n-2)!}{(n+1)\times n\times(n-1)\times(n-2)!} = \frac{1}{n(n+1)(n-1)}$$

Write the following series using sigma/summation notation with the lower limit of summation $k=2$. **SCORE: ___ / 3 POINTS**

$$\frac{4\times 1^2}{3} - \frac{8\times 2^2}{4} + \frac{16\times 3^2}{5} - \frac{32\times 4^2}{6}$$

$$= \sum_{k=2}^{5} (-1)^k \frac{2^k (k-1)^2}{k+1}$$

Suppose that $a_0 = 0$ and $a_1 = 4$ and $a_k = 6a_{k-1} - 5a_{k-2}$ for all integers $k \ge 2$. **SCORE: ___ / 8 POINTS**

Prove that $a_n = 5^n - 1$ for all non-negative integers n.

BASIS STEP: $n=0$: $a_0 = 0 = 5^0 - 1$

$n=1$: $a_1 = 4 = 5^1 - 1$

INDUCTIVE STEP: Assume $a_n = 5^n - 1$ for $n = 0, 1, \ldots, k$ for some particular but arbitrary non-negative integer $k \ge 1$.

So, $k-1 \ge 0$ and $k-1 \le k$.

a_{k+1}

$= 6a_k - 5a_{k-1}$

$= 6(5^k - 1) - 5(5^{k-1} - 1)$

$= 6\times 5^k - 6 - 5^k + 5$

$= 5\times 5^k - 1$

$= 5^{k+1} - 1$

By strong induction, $a_n = 5^n - 1$ for all non-negative integers n.

Math 22 (9:30am – 10:20am)
Quiz 5 Version H
Fri Nov 4, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 24 POINTS

Prove that $5^n + 9 < 6^n$ for all integers $n \geq 2$ by mathematical induction. **SCORE: ___ / 9 POINTS**

BASIS STEP: $n = 2$: $5^2 + 9 = 34 < 36 = 6^2$

INDUCTIVE STEP: Assume $5^k + 9 < 6^k$ for some particular but arbitrary integer $k \geq 2$.

So, $5^k < 6^k - 9$

$$\begin{aligned} &5^{k+1} + 9 \\ &= 5 \times 5^k + 9 \\ &< 5 \times (6^k - 9) + 9 \\ &= 5 \times 6^k - 45 + 9 \\ &= 5 \times 6^k - 36 \\ &< 6 \times 6^k - 36 \\ &< 6 \times 6^k \\ &= 6^{k+1} \end{aligned}$$

OR

$$\begin{aligned} &6^{k+1} \\ &= 6 \times 6^k \\ &> 6 \times (5^k + 9) \\ &> 5 \times (5^k + 9) \\ &= 5 \times 5^k + 45 \\ &= 5^{k+1} + 45 \\ &> 5^{k+1} + 9 \end{aligned}$$

By mathematical induction, $5^n + 9 < 6^n$ for all integers $n \geq 2$.

Write the following series using sigma/summation notation with the lower limit of summation $k = 2$. **SCORE: ___ / 3 POINTS**

$$\frac{4 \times 1^2}{3} - \frac{8 \times 2^2}{4} + \frac{16 \times 3^2}{5} - \frac{32 \times 4^2}{6}$$

$$= \sum_{k=2}^{5} (-1)^k \frac{2^k (k-1)^2}{k+1}$$

Suppose that $a_0 = 0$ and $a_1 = 4$ and $a_k = 6a_{k-1} - 5a_{k-2}$ for all integers $k \geq 2$. **SCORE: ___ / 8 POINTS**

Prove that $a_n = 5^n - 1$ for all non-negative integers n.

BASIS STEP: $n = 0$: $a_0 = 0 = 5^0 - 1$

$n = 1$: $a_1 = 4 = 5^1 - 1$

INDUCTIVE STEP: Assume $a_n = 5^n - 1$ for $n = 0, 1, \ldots, k$ for some particular but arbitrary non-negative integer $k \geq 1$.

So, $k - 1 \geq 0$ and $k - 1 \leq k$.

$$\begin{aligned}
&a_{k+1} \\
&= 6a_k - 5a_{k-1} \\
&= 6(5^k - 1) - 5(5^{k-1} - 1) \\
&= 6 \times 5^k - 6 - 5^k + 5 \\
&= 5 \times 5^k - 1 \\
&= 5^{k+1} - 1
\end{aligned}$$

By strong induction, $a_n = 5^n - 1$ for all non-negative integers n.

Evaluate $\binom{8}{5}$. **SCORE: ___ / 2 POINTS**

$$\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6 \times 5!}{5!3!} = \frac{8 \times 7 \times 6}{3!} = \frac{8 \times 7 \times 6}{6} = 56$$

Simplify $\frac{(n-2)!}{(n+1)!}$. **SCORE: ___ / 2 POINTS**

$$\frac{(n-2)!}{(n+1)!} = \frac{(n-2)!}{(n+1) \times n \times (n-1) \times (n-2)!} = \frac{1}{n(n+1)(n-1)}$$

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ___________**
Quiz 5 Version O
Fri Nov 4, 2011

SCORE: ___ / 24 POINTS

Evaluate $\binom{7}{3}$. **SCORE: ___ / 2 POINTS**

$$\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3!4!} = \frac{7 \times 6 \times 5}{3!} = \frac{7 \times 6 \times 5}{6} = 35$$

Suppose that $a_0 = 0$ and $a_1 = 4$ and $a_k = 6a_{k-1} - 5a_{k-2}$ for all integers $k \geq 2$. **SCORE: ___ / 8 POINTS**
Prove that $a_n = 5^n - 1$ for all non-negative integers n.

BASIS STEP: $n = 0$: $a_0 = 0 = 5^0 - 1$

$n = 1$: $a_1 = 4 = 5^1 - 1$

INDUCTIVE STEP: Assume $a_n = 5^n - 1$ for $n = 0, 1, \ldots, k$ for some particular but arbitrary non-negative integer $k \geq 1$.

So, $k - 1 \geq 0$ and $k - 1 \leq k$.

a_{k+1}
$= 6a_k - 5a_{k-1}$
$= 6(5^k - 1) - 5(5^{k-1} - 1)$
$= 6 \times 5^k - 6 - 5^k + 5$
$= 5 \times 5^k - 1$
$= 5^{k+1} - 1$

By strong induction, $a_n = 5^n - 1$ for all non-negative integers n.

Simplify $\dfrac{(n-1)!}{(n+2)!}$. **SCORE: ___ / 2 POINTS**

$$\frac{(n-1)!}{(n+2)!} = \frac{(n-1)!}{(n+2)\times(n+1)\times n\times(n-1)!} = \frac{1}{n(n+2)(n+1)}$$

Write the following series using sigma/summation notation with the lower limit of summation $k = 3$. **SCORE: ___ / 3 POINTS**

$$\frac{3\times1^2}{4} - \frac{4\times2^2}{8} + \frac{5\times3^2}{16} - \frac{6\times4^2}{32}$$

$$= \sum_{k=3}^{6} (-1)^{k-1} \frac{k(k-2)^2}{2^{k-1}}$$

Prove that $5^n + 9 < 6^n$ for all integers $n \ge 2$ by mathematical induction. **SCORE: ___ / 9 POINTS**

BASIS STEP: $n = 2$: $5^2 + 9 = 34 < 36 = 6^2$

INDUCTIVE STEP: Assume $5^k + 9 < 6^k$ for some particular but arbitrary integer $k \ge 2$.

So, $5^k < 6^k - 9$

$5^{k+1} + 9$	OR	6^{k+1}
$= 5\times5^k + 9$		$= 6\times6^k$
$< 5\times(6^k - 9) + 9$		$> 6\times(5^k + 9)$
$= 5\times6^k - 45 + 9$		$> 5\times(5^k + 9)$
$= 5\times6^k - 36$		$= 5\times5^k + 45$
$< 6\times6^k - 36$		$= 5^{k+1} + 45$
$< 6\times6^k$		$> 5^{k+1} + 9$
$= 6^{k+1}$		

By mathematical induction, $5^n + 9 < 6^n$ for all integers $n \ge 2$.

Math 22 (9:30am – 10:20am)
Quiz 5 Version W
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NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 24 POINTS

Suppose that $a_0 = 0$ and $a_1 = 4$ and $a_k = 6a_{k-1} - 5a_{k-2}$ for all integers $k \ge 2$. **SCORE: ___ / 8 POINTS**

Prove that $a_n = 5^n - 1$ for all non-negative integers n.

BASIS STEP: $n = 0$: $a_0 = 0 = 5^0 - 1$

$n = 1$: $a_1 = 4 = 5^1 - 1$

INDUCTIVE STEP: Assume $a_n = 5^n - 1$ for $n = 0, 1, \ldots, k$ for some particular but arbitrary non-negative integer $k \ge 1$.

So, $k - 1 \ge 0$ and $k - 1 \le k$.

$$\begin{aligned}
&a_{k+1} \\
&= 6a_k - 5a_{k-1} \\
&= 6(5^k - 1) - 5(5^{k-1} - 1) \\
&= 6 \times 5^k - 6 - 5^k + 5 \\
&= 5 \times 5^k - 1 \\
&= 5^{k+1} - 1
\end{aligned}$$

By strong induction, $a_n = 5^n - 1$ for all non-negative integers n.

Write the following series using sigma/summation notation with the lower limit of summation $k = 3$. **SCORE: ___ / 3 POINTS**

$$\frac{3 \times 1^2}{4} - \frac{4 \times 2^2}{8} + \frac{5 \times 3^2}{16} - \frac{6 \times 4^2}{32}$$

$$= \sum_{k=2}^{5} (-1)^k \frac{2^k (k-1)^2}{k+1}$$

Prove that $5^n + 9 < 6^n$ for all integers $n \geq 2$ by mathematical induction. **SCORE: ___ / 9 POINTS**

BASIS STEP: $n = 2$: $5^2 + 9 = 34 < 36 = 6^2$

INDUCTIVE STEP: Assume $5^k + 9 < 6^k$ for some particular but arbitrary integer $k \geq 2$.

So, $5^k < 6^k - 9$

$$\begin{aligned} & 5^{k+1} + 9 \\ &= 5 \times 5^k + 9 \\ &< 5 \times (6^k - 9) + 9 \\ &= 5 \times 6^k - 45 + 9 \\ &= 5 \times 6^k - 36 \\ &< 6 \times 6^k - 36 \\ &< 6 \times 6^k \\ &= 6^{k+1} \end{aligned}$$

OR

$$\begin{aligned} & 6^{k+1} \\ &= 6 \times 6^k \\ &> 6 \times (5^k + 9) \\ &> 5 \times (5^k + 9) \\ &= 5 \times 5^k + 45 \\ &= 5^{k+1} + 45 \\ &> 5^{k+1} + 9 \end{aligned}$$

By mathematical induction, $5^n + 9 < 6^n$ for all integers $n \geq 2$.

Evaluate $\binom{7}{3}$. **SCORE: ___ / 2 POINTS**

$$\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3!4!} = \frac{7 \times 6 \times 5}{3!} = \frac{7 \times 6 \times 5}{6} = 35$$

Simplify $\frac{(n-1)!}{(n+2)!}$. **SCORE: ___ / 2 POINTS**

$$\frac{(n-1)!}{(n+2)!} = \frac{(n-1)!}{(n+2) \times (n+1) \times n \times (n-1)!} = \frac{1}{n(n+2)(n+1)}$$