Math 22 (9:30am – 10:20am)
Quiz 6 Version W
Fri Nov 18, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ____________

SCORE: ___ / 24 POINTS

In an element argument, the statement "$x \in A \cap B$" translates into "$x \in A$ and $x \in B$". **SCORE: ___ / 3 POINTS**
Translate the following statements **<u>without using set complements</u>**. Simplify fully using logical equivalences where possible.

"$x \notin A \cup B$" translates into "not ($x \in A$ or $x \in B$)" ie. "$x \notin A$ and $x \notin B$"

"$x \notin A - B$" translates into "not ($x \in A$ and $x \notin B$)" ie. "$x \notin A$ or $x \in B$"

"$X \in \wp(A)$" translates into "$X \subseteq A$"

Prove the following statement **using an element argument**. **<u>Do not use any set identities.</u>** **SCORE: ___ / 7 POINTS**

$A \cap (A \cup B) = A$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

[NEED TO SHOW: $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$
ie. $\forall x \in A \cap (A \cup B), \quad x \in A$ and $\forall x \in A, \quad x \in A \cap (A \cup B)$]

Let $x \in A \cap (A \cup B)$.
So, $x \in A$ and $x \in A \cup B$.
So, $x \in A$.
So, $A \cap (A \cup B) \subseteq A$.

Let $x \in A$.
So, $x \in A$ or $x \in B$.
So, $x \in A \cup B$.
And also $x \in A$, so, $x \in A \cap (A \cup B)$.
So, $A \subseteq A \cap (A \cup B)$.

Since $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$,
therefore $A \cap (A \cup B) = A$.

Let B be a Boolean algebra with operations $+$ and $\cdot$. **SCORE: ___ / 7 POINTS**
Prove the following statement **without using De Morgan's Laws**. State the name of the law that justifies each step.

$(a \cdot b) + (\overline{a} + \overline{b}) = 1$ for all $a, b \in B$

PROOF: Let $a, b \in B$.

$$
\begin{aligned}
& (a \cdot b) + (\overline{a} + \overline{b}) & \\
&= (\overline{a} + \overline{b}) + (a \cdot b) && \text{COMMUTATIVE} \\
&= ((\overline{a} + \overline{b}) + a) \cdot ((\overline{a} + \overline{b}) + b) && \text{DISTRIBUTIVE} \\
&= ((\overline{b} + \overline{a}) + a) \cdot ((\overline{a} + \overline{b}) + b) && \text{COMMUTATIVE} \\
&= (\overline{b} + (\overline{a} + a)) \cdot (\overline{a} + (\overline{b} + b)) && \text{ASSOCIATIVE} \\
&= (\overline{b} + (a + \overline{a})) \cdot (\overline{a} + (b + \overline{b})) && \text{COMMUTATIVE} \\
&= (\overline{b} + 1) \cdot (\overline{a} + 1) && \text{COMPLEMENT} \\
&= 1 \cdot 1 && \text{UNIVERSAL BOUNDS} \\
&= 1 && \text{IDENTITY}
\end{aligned}
$$

Prove the following statement **using set identities**. State the name of the law that justifies each step. **SCORE: ___ / 7 POINTS**
NOTE: The law that translates a set difference into $\cup$ and/or $\cap$ and/or ^{C} is called the Set Difference Law.

$A \cap B = A - (A - B)$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

$$
\begin{aligned}
& A - (A - B) & \\
&= A \cap (A \cap B^C)^C && \text{SET DIFFERENCE} \\
&= A \cap (A^C \cup (B^C)^C) && \text{DE MORGAN'S LAW} \\
&= A \cap (A^C \cup B) && \text{DOUBLE COMPLEMENT} \\
&= (A \cap A^C) \cup (A \cap B) && \text{DISTRIBUTIVE} \\
&= \varnothing \cup (A \cap B) && \text{UNIVERSAL BOUNDS} \\
&= (A \cap B) \cup \varnothing && \text{COMMUTATIVE} \\
&= A \cap B && \text{IDENTITY}
\end{aligned}
$$

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ___________**
Quiz 6 Version A
Fri Nov 18, 2011

SCORE: ___ / 24 POINTS

In an element argument, the statement "$x \in A \cap B$" translates into "$x \in A$ and $x \in B$". **SCORE: ___ / 3 POINTS**
Translate the following statements **without using set complements**. Simplify fully using logical equivalences where possible.

"$x \notin A - B$" translates into "not ($x \in A$ and $x \notin B$)" ie. "$x \notin A$ or $x \in B$"

"$X \in \wp(A)$" translates into "$X \subseteq A$"

"$x \notin A \cup B$" translates into "not ($x \in A$ or $x \in B$)" ie. "$x \notin A$ and $x \notin B$"

Prove the following statement **using set identities**. State the name of the law that justifies each step. **SCORE: ___ / 7 POINTS**
NOTE: The law that translates a set difference into $\cup$ and/or $\cap$ and/or ^{C} is called the Set Difference Law.

$A \cap B = A - (A - B)$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

$A - (A - B)$
$= A \cap (A \cap B^C)^C$ SET DIFFERENCE
$= A \cap (A^C \cup (B^C)^C)$ DE MORGAN'S LAW
$= A \cap (A^C \cup B)$ DOUBLE COMPLEMENT
$= (A \cap A^C) \cup (A \cap B)$ DISTRIBUTIVE
$= \varnothing \cup (A \cap B)$ UNIVERSAL BOUNDS
$= (A \cap B) \cup \varnothing$ COMMUTATIVE
$= A \cap B$ IDENTITY

Prove the following statement **using an element argument**. **Do not use any set identities.** **SCORE: ___ / 7 POINTS**

$A \cap (A \cup B) = A$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

[NEED TO SHOW: $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$

ie. $\forall x \in A \cap (A \cup B), \quad x \in A$ and $\forall x \in A, \quad x \in A \cap (A \cup B)$]

Let $x \in A \cap (A \cup B)$.
So, $x \in A$ and $x \in A \cup B$.
So, $x \in A$.
So, $A \cap (A \cup B) \subseteq A$.

Let $x \in A$.
So, $x \in A$ or $x \in B$.
So, $x \in A \cup B$.
And also $x \in A$, so, $x \in A \cap (A \cup B)$.
So, $A \subseteq A \cap (A \cup B)$.

Since $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$,
therefore $A \cap (A \cup B) = A$.

Let B be a Boolean algebra with operations $+$ and $\cdot$. **SCORE: ___ / 7 POINTS**
Prove the following statement **without using De Morgan's Laws**. State the name of the law that justifies each step.

$(a \cdot b) + (\overline{a} + \overline{b}) = 1$ for all $a, b \in B$

PROOF: Let $a, b \in B$.

$(a \cdot b) + (\overline{a} + \overline{b})$	
$= (\overline{a} + \overline{b}) + (a \cdot b)$	COMMUTATIVE
$= ((\overline{a} + \overline{b}) + a) \cdot ((\overline{a} + \overline{b}) + b)$	DISTRIBUTIVE
$= ((\overline{b} + \overline{a}) + a) \cdot ((\overline{a} + \overline{b}) + b)$	COMMUTATIVE
$= (\overline{b} + (\overline{a} + a)) \cdot (\overline{a} + (\overline{b} + b))$	ASSOCIATIVE
$= (\overline{b} + (a + \overline{a})) \cdot (\overline{a} + (b + \overline{b}))$	COMMUTATIVE
$= (\overline{b} + 1) \cdot (\overline{a} + 1)$	COMPLEMENT
$= 1 \cdot 1$	UNIVERSAL BOUNDS
$= 1$	IDENTITY

Math 22 (9:30am – 10:20am) **NAME YOU ASKED TO BE CALLED IN CLASS: ___________**
Quiz 6 Version X
Fri Nov 18, 2011

SCORE: ___ / 24 POINTS

In an element argument, the statement "$x \in A \cap B$" translates into "$x \in A$ and $x \in B$". **SCORE: ___ / 3 POINTS**
Translate the following statements **without using set complements**. Simplify fully using logical equivalences where possible.

"$X \in \wp(A)$" translates into "$X \subseteq A$"

"$x \notin A \cup B$" translates into "not ($x \in A$ or $x \in B$)" ie. "$x \notin A$ and $x \notin B$"

"$x \notin A - B$" translates into "not ($x \in A$ and $x \notin B$)" ie. "$x \notin A$ or $x \in B$"

Let B be a Boolean algebra with operations $+$ and $\cdot$. **SCORE: ___ / 7 POINTS**
Prove the following statement **without using De Morgan's Laws**. State the name of the law that justifies each step.

$(a \cdot b) + (\bar{a} + \bar{b}) = 1$ for all $a, b \in B$

PROOF: Let $a, b \in B$.

$(a \cdot b) + (\bar{a} + \bar{b})$

$= (\bar{a} + \bar{b}) + (a \cdot b)$ COMMUTATIVE

$= ((\bar{a} + \bar{b}) + a) \cdot ((\bar{a} + \bar{b}) + b)$ DISTRIBUTIVE

$= ((\bar{b} + \bar{a}) + a) \cdot ((\bar{a} + \bar{b}) + b)$ COMMUTATIVE

$= (\bar{b} + (\bar{a} + a)) \cdot (\bar{a} + (\bar{b} + b))$ ASSOCIATIVE

$= (\bar{b} + (a + \bar{a})) \cdot (\bar{a} + (b + \bar{b}))$ COMMUTATIVE

$= (\bar{b} + 1) \cdot (\bar{a} + 1)$ COMPLEMENT

$= 1 \cdot 1$ UNIVERSAL BOUNDS

$= 1$ IDENTITY

Prove the following statement **using set identities**. State the name of the law that justifies each step. **SCORE: ___ / 7 POINTS**

NOTE: The law that translates a set difference into $\cup$ and/or $\cap$ and/or ^{C} is called the Set Difference Law.

$A \cap B = A - (A - B)$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

$A - (A - B)$

$= A \cap (A \cap B^{C})^{C}$ SET DIFFERENCE

$= A \cap (A^{C} \cup (B^{C})^{C})$ DE MORGAN'S LAW

$= A \cap (A^{C} \cup B)$ DOUBLE COMPLEMENT

$= (A \cap A^{C}) \cup (A \cap B)$ DISTRIBUTIVE

$= \varnothing \cup (A \cap B)$ UNIVERSAL BOUNDS

$= (A \cap B) \cup \varnothing$ COMMUTATIVE

$= A \cap B$ IDENTITY

Prove the following statement **using an element argument**. **Do not use any set identities.** **SCORE: ___ / 7 POINTS**

$A \cap (A \cup B) = A$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

[NEED TO SHOW: $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$

ie. $\forall x \in A \cap (A \cup B), \quad x \in A$ and $\forall x \in A, \quad x \in A \cap (A \cup B)$]

Let $x \in A \cap (A \cup B)$.
So, $x \in A$ and $x \in A \cup B$.
So, $x \in A$.
So, $A \cap (A \cup B) \subseteq A$.

Let $x \in A$.
So, $x \in A$ or $x \in B$.
So, $x \in A \cup B$.
And also $x \in A$, so, $x \in A \cap (A \cup B)$.
So, $A \subseteq A \cap (A \cup B)$.

Since $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$,
therefore $A \cap (A \cup B) = A$.

Math 22 (9:30am – 10:20am)
Quiz 6 Version Y
Fri Nov 18, 2011

NAME YOU ASKED TO BE CALLED IN CLASS: ___________

SCORE: ___ / 24 POINTS

Prove the following statement **using set identities**. State the name of the law that justifies each step. **SCORE: ___ / 7 POINTS**

NOTE: The law that translates a set difference into $\cup$ and/or $\cap$ and/or ^{C} is called the Set Difference Law.

$A \cap B = A - (A - B)$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

$$
\begin{array}{ll}
A-(A-B) & \\
= A \cap (A \cap B^C)^C & \text{SET DIFFERENCE} \\
= A \cap (A^C \cup (B^C)^C) & \text{DE MORGAN'S LAW} \\
= A \cap (A^C \cup B) & \text{DOUBLE COMPLEMENT} \\
= (A \cap A^C) \cup (A \cap B) & \text{DISTRIBUTIVE} \\
= \varnothing \cup (A \cap B) & \text{UNIVERSAL BOUNDS} \\
= (A \cap B) \cup \varnothing & \text{COMMUTATIVE} \\
= A \cap B & \text{IDENTITY}
\end{array}
$$

Let B be a Boolean algebra with operations $+$ and $\cdot$. **SCORE: ___ / 7 POINTS**

Prove the following statement **without using De Morgan's Laws**. State the name of the law that justifies each step.

$(a \cdot b) + (\overline{a} + \overline{b}) = 1$ for all $a, b \in B$

PROOF: Let $a, b \in B$.

$$
\begin{array}{ll}
(a \cdot b) + (\overline{a} + \overline{b}) & \\
= (\overline{a} + \overline{b}) + (a \cdot b) & \text{COMMUTATIVE} \\
= ((\overline{a} + \overline{b}) + a) \cdot ((\overline{a} + \overline{b}) + b) & \text{DISTRIBUTIVE} \\
= ((\overline{b} + \overline{a}) + a) \cdot ((\overline{a} + \overline{b}) + b) & \text{COMMUTATIVE} \\
= (\overline{b} + (\overline{a} + a)) \cdot (\overline{a} + (\overline{b} + b)) & \text{ASSOCIATIVE} \\
= (\overline{b} + (a + \overline{a})) \cdot (\overline{a} + (b + \overline{b})) & \text{COMMUTATIVE} \\
= (\overline{b} + 1) \cdot (\overline{a} + 1) & \text{COMPLEMENT} \\
= 1 \cdot 1 & \text{UNIVERSAL BOUNDS} \\
= 1 & \text{IDENTITY}
\end{array}
$$

Prove the following statement **using an element argument**. **Do not use any set identities.** **SCORE: ___ / 7 POINTS**

$A \cap (A \cup B) = A$ for all sets A, B which are subsets of a universal set U

PROOF: Let $A, B \subseteq U$.

[NEED TO SHOW: $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$

ie. $\forall x \in A \cap (A \cup B),\quad x \in A$ and $\forall x \in A,\quad x \in A \cap (A \cup B)$]

Let $x \in A \cap (A \cup B)$.
So, $x \in A$ and $x \in A \cup B$.
So, $x \in A$.
So, $A \cap (A \cup B) \subseteq A$.

Let $x \in A$.
So, $x \in A$ or $x \in B$.
So, $x \in A \cup B$.
And also $x \in A$, so, $x \in A \cap (A \cup B)$.
So, $A \subseteq A \cap (A \cup B)$.

Since $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$,
therefore $A \cap (A \cup B) = A$.

In an element argument, the statement "$x \in A \cap B$" translates into "$x \in A$ and $x \in B$". **SCORE: ___ / 3 POINTS**
Translate the following statements **without using set complements**. Simplify fully using logical equivalences where possible.

"$x \notin A \cup B$" translates into "not ($x \in A$ or $x \in B$)" ie. "$x \notin A$ and $x \notin B$"

"$x \notin A - B$" translates into "not ($x \in A$ and $x \notin B$)" ie. "$x \notin A$ or $x \in B$"

"$X \in \wp(A)$" translates into "$X \subseteq A$"