Math 114 (8:30am – 9:20am)
Midterm 1 Version O
Wed Apr 28, 2010

NAME YOU AKSED TO BE CALLED: _____

SCORE: _____ / 140 POINTS

- ALL PROBLEMS MUST BE SOLVED ALGEBRAICALLY TO EARN CREDIT (NO GUESS & CHECK)
- PUT A BOX AROUND EACH FINAL ANSWER
- SHOW COMPLETE AND PROPER WORK TO EARN FULL CREDIT

Solve: The height of a chocolate pyramid varies directly as the volume of chocolate used, and inversely as the area of the base of the pyramid. 5 liters of chocolate can be molded into a 0.6 meter tall pyramid with a base of area 250 cm^2. How tall is a pyramid with a base of area 400 cm^2 molded from 4 liters of chocolate ?

SCORE: ___ / 15 POINTS

FOR FULL CREDIT, YOU MUST IDENTIFY WHAT ALL YOUR VARIABLES REPRESENT, FIND THE SPECIFIC EQUATION CONNECTING THEM, AND SUMMARIZE YOUR FINAL ANSWER IN A SENTENCE USING THE CORRECT UNITS OF MEASUREMENT.

1 [H = HEIGHT
V = VOLUME
A = AREA OF BASE

4 [$H = \frac{kV}{A}$

2 [$0.6 = \frac{k(5)}{250}$

2 [$k = 30$

$H = \frac{30V}{A}$] 2

$H = \frac{30(4)}{400}$] 1

$H = 0.3$] 1

[4L OF CHOCOLATE WILL MAKE A PYRAMID 0.3 m TALL WITH A BASE OF 400 cm^2] 1

Subtract and simplify: $\frac{x-5}{x^2+5x+4} - \frac{x-2}{x^2+6x+8}$

SCORE: ___ / 15 POINTS

$= \left[\frac{x-5}{(x+1)(x+4)} \cdot \frac{x+2}{x+2}\right]^1 - \left[\frac{x-2}{(x+2)(x+4)} \cdot \frac{x+1}{x+1}\right]$ 1

$= \frac{(x^2-3x-10) - (x^2-x-2)}{(x+1)(x+4)(x+2)}$ 1 2 2

$= \frac{-2x-8}{(x+1)(x+4)(x+2)}$ 3 2

$= \frac{-2(x+4)}{(x+1)(x+4)(x+2)}$ $\frac{1}{2}$

$= \boxed{\frac{-2}{(x+1)(x+2)}}$ $1\frac{1}{2}$

Solve: A number divided by eight is equal to nine divided by six less than that number. Find the number. **CHECK YOUR ANSWER(S).**

SCORE: ___ / 15 POINTS

$\frac{x}{8} = \frac{9}{x-6}$ 5

$x^2 - 6x = 72$ 2

$x^2 - 6x - 72 = 0$ 1

$(x-12)(x+6) = 0$ 2

$x = 12, -6$ 2

CHECK: $\frac{12}{8} = \frac{3}{2}$

$\frac{9}{12-6} = \frac{9}{6} = \frac{3}{2}$ ✓] 1

$\frac{-6}{8} = \frac{-3}{4}$

$\frac{9}{-6-6} = \frac{9}{-12} = -\frac{3}{4}$ ✓] 1

+1 FOR IDENTIFYING BOTH SOLUTIONS AS CORRECT AFTER CHECK

Find the equation of the vertical asymptote of $y = \frac{5+7x}{14x-8}$.

SCORE: ___ / 8 POINTS

$14x - 8 = 0$ 1½

$x = \frac{8}{14} = \frac{4}{7}$

$x = \frac{4}{7}$ 1 1½

Find the equation of the horizontal asymptote of $y = \frac{5+7x}{14x-8}$.

AS $x \to \pm\infty$, $y \approx \frac{7x}{14x} = \frac{1}{2}$ 1½

$y = \frac{1}{2}$ 1 1½

Simplify: $\frac{3x^2 + 20x - 7}{4x^2 + 31x + 21}$ **SPECIFY ANY RESTRICTIONS.**

SCORE: ___ / 15 POINTS

$= \frac{(3x-1)(x+7)}{(4x+3)(x+7)}$ 5 (3 IF COEFFICIENTS OK BUT SIGNS WRONG) 5 (3 ")

$= \frac{3x-1}{4x+3}$ IF $x \neq -7$

2 3

Solve for x: $\dfrac{4}{x^2+2x-3}+\dfrac{2}{x^2+6x+9}=\dfrac{1}{x-1}$ **CHECK YOUR ANSWER(S)** **SCORE: ___ / 15 POINTS**

$(x-1)(x+3)^2\left[\dfrac{4}{(x+3)(x-1)}+\dfrac{2}{(x+3)^2}\right]=\left[\dfrac{1}{x-1}\right](x-1)(x+3)^2$ 4

$4(x+3)+2(x-1)=(x+3)^2$ ½ ½ ½

$4x+12+2x-2=x^2+6x+9$ ½ ½ 1

$6x+10=x^2+6x+9$ ½

$0=x^2-1$ 1

$0=(x+1)(x-1)$ ½

$x=-1$ 1 ½

CHECK: $x=-1$

$\dfrac{4}{-4}+\dfrac{2}{2^2}=-\dfrac{1}{2}$ 1

$\dfrac{1}{-2}=-\dfrac{1}{2}$ ✓

$x=1$ 1

$\dfrac{4}{0}$ ✗

+1 FOR IDENTIFYING −1 AS THE ONLY CORRECT SOLUTION

Simplify: $\dfrac{1-\dfrac{3}{x-5}}{\dfrac{4}{x-5}-\dfrac{8}{x-2}}$ **SCORE: ___ / 15 POINTS**

$=\dfrac{1-\frac{3}{x-5}}{\frac{4}{x-5}-\frac{8}{x-2}}\cdot\dfrac{(x-5)(x-2)}{(x-5)(x-2)}$ 4

$=\dfrac{(x-5)(x-2)-3(x-2)}{4(x-2)-8(x-5)}$ 1 1 1

$=\dfrac{(x-2)(x-5-3)}{4x-8-8x+40}$ 1 1

$=\dfrac{(x-2)(x-8)}{-4x+32}$ 2 1

$=\dfrac{(x-2)(x-8)}{-4(x-8)}$ 1

$=-\dfrac{x-2}{4}$ 1

OR

$=\dfrac{\frac{(x-5)-3}{x-5}}{\frac{4(x-2)-8(x-5)}{(x-5)(x-2)}}$ 2 2

$=\dfrac{\frac{x-8}{x-5}}{\frac{4x-8-8x+40}{(x-5)(x-2)}}$ 1 1

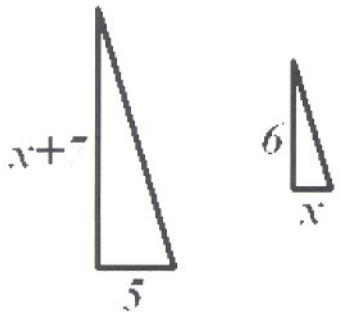
$=\dfrac{\frac{x-8}{x-5}}{\frac{-4x+32}{(x-5)(x-2)}}$ 1 1

$=\dfrac{x-8}{x-5}\cdot\dfrac{(x-5)(x-2)}{-4x+32}$ 3

$=\dfrac{(x-8)(x-2)}{-4(x-8)}=-\dfrac{x-2}{4}$ 1 1

Solve for x in the following similar triangles:

SCORE: ___ / 12 POINTS



$\frac{x+7}{6} = \frac{5}{x}$, 5

$x^2+7x=30$, 2

$x^2+7x-30=0$, 1

$(x+10)(x-3)=0$, 2

$x=-10, \boxed{3}$, $1\frac{1}{2}$

+$1\frac{1}{2}$ FOR IDENTIFYING 3 AS THE ONLY CORRECT SOLUTION

Divide and simplify: $\frac{18x^2-32}{15-6x} \div \frac{12x^2+16x}{10x^2-25x}$

SCORE: ___ / 15 POINTS

$= \frac{18x^2-32}{15-6x} \cdot \frac{10x^2-25x}{12x^2+16x}$, 4

$= \frac{2(9x^2-16)}{-3(2x-5)} \cdot \frac{5x(2x-5)}{4x(3x+4)}$, 1 , 1 , 1 , 1

$= \frac{5(3x+4)(3x-4)}{-6(3x+4)} = \boxed{-\frac{5(3x-4)}{6}}$, 2

Subtract and simplify: $\frac{4x^2-2x-1}{x^2+5x-6} - \frac{x^2-3x+3}{x^2+5x-6}$

SCORE: ___ / 15 POINTS

$= \frac{3x^2+x-4}{(x+6)(x-1)}$, 9 , 2

$= \frac{(3x+4)(x-1)}{(x+6)(x-1)}$, 2

$= \boxed{\frac{3x+4}{x+6}}$, 2