Math 1A (7:30am – 8:20am)
Midterm 1 Version B
Wed Apr 27, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER WORK (SO I CAN TELL HOW YOU GOT YOUR ANSWERS)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem. **SCORE: ___ / 10 POINTS**
Use complete sentences and proper algebra & English as shown in class.

IF f IS CONTINUOUS AND $g(x)=\int_a^x f(t)\,dt$, THEN $g'(x)=f(x)$

IF f IS CONTINUOUS ON $[a,b]$ AND F IS ANY ANTIDERIVATIVE OF f THEN $\int_a^b f(x)\,dx = F(b)-F(a)$

IF F' IS CONTINUOUS ON $[a,b]$ THEN $\int_a^b F'(x)\,dx = F(b)-F(a)$

If f and g are continuous functions such that $f(x) > g(x)$ for all x, and $\int_a^b f(x)\,dx < \int_a^b g(x)\,dx$, **SCORE: ___ / 5 POINTS**

what can you conclude about this situation ? **Explain very briefly.**

IF $f>g$ AND $b>a$ THEN $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$ 2

SINCE $f>g$ BUT $\int_a^b f(x)\,dx < \int_a^b g(x)\,dx$

THEREFORE $b<a$ 3

The acceleration of an object at time t (in hours) is given by $a(t)=t^2 \tanh t$ miles per hour^2, where $t=0$ corresponds to noon. At three hours before noon, the object's velocity was 11 miles per hour. Find the object's velocity at three hours after noon. **SCORE: ___ / 20 POINTS**

$\underbrace{v(3)-v(-3)}_{4} = \int_{-3}^{3} v'(t)\,dt = \underbrace{\int_{-3}^{3} a(t)\,dt}_{2} = \underbrace{\int_{-3}^{3} t^2 \tanh t\,dt}_{4}$

$(-t)^2 \tanh(-t)$ 2
$= t^2(-\tanh t)$
$= -t^2 \tanh t$ 2

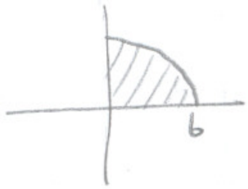
SINCE THE INTEGRAND IS ODD, THE INTEGRAL IS 0 (2)

2 $v(3)-v(-3)=0 \Rightarrow v(3)=v(-3)=11$ MPH 2

Let $h(m) = \int_{m^2-3}^{3-m} \sqrt{36-y^2}\, dy$. **SCORE: ___ / 30 POINTS**

8 [a] Find $h(3)$.

$$\underbrace{\int_6^0 \sqrt{36-y^2}\,dy}_{2} = \underbrace{-\int_0^6 \sqrt{36-y^2}\,dy}_{1}$$



$$= \underbrace{-\tfrac{1}{4}(\pi \cdot 6^2)}_{4} = \underbrace{-9\pi}_{1}$$

4 [b] Find $h(2)$.

$$\underbrace{\int_1^1 \sqrt{36-y^2}\,dy}_{2} = \underbrace{0}_{2}$$

18 [c] Find $h'(1)$.

$$h'(m) = \frac{d}{dm}\left[\int_0^{3-m} \sqrt{36-y^2}\,dy - \int_0^{m^2-3} \sqrt{36-y^2}\,dy\right] \quad 2 \quad 2 \quad 2$$

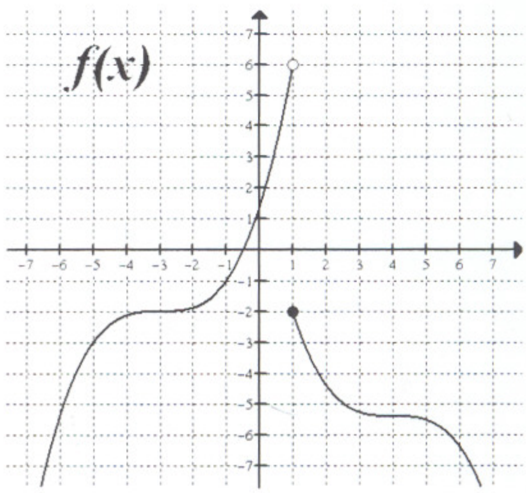
$$= \underbrace{\sqrt{36-(3-m)^2}}_{2} \cdot \underbrace{(-1)}_{2} - \underbrace{\sqrt{36-(m^2-3)^2}}_{2} \cdot \underbrace{2m}_{2}$$

$$\underbrace{h'(1) = -\sqrt{32} - 2\sqrt{32}}_{2}$$

$$= -3\sqrt{32}$$

$$= \underbrace{-12\sqrt{2}}_{2}$$

The graph of $f(x)$ is shown on the right. Let $g(x) = \int_2^x f(t)\, dt$. **SCORE: ___ / 25 POINTS**



6 [a] At what value(s) of x does g have an inflection point?
Explain very briefly.

$\underbrace{g' = f}_{2}$ CHANGES FROM INCREASING TO DECREASING 2

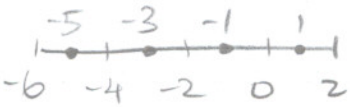
AT $\underbrace{x = 1}_{2}$

2 (ONLY IF EXPLANATION PROVIDED)

7 [b] Estimate $g(-6)$ using the Midpoint Rule with 4 equal subintervals.

$$g(-6) = \underbrace{\int_2^{-6} f(t)\,dt}_{2}$$

$$= \underbrace{-\int_{-6}^{2} f(t)\,dt}_{1}$$



$$\Delta x = \frac{2-(-6)}{4} = 2$$

$$\approx -\big(\underbrace{f(-5)+f(-3)+f(-1)+f(1)}_{8}\big)\,2$$

$$= -4\big(\underbrace{-3 + -2 + -1 + -2}\big)\underbrace{2}_{3} = \underbrace{16}_{1}$$

Find $\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 1}}\,dx$. **SCORE: ___ / 20 POINTS**

$u = \cosh^2 x$ 4

$du = 2\cosh x \sinh x\, dx$ 2

$= \sinh 2x\, dx$ 2

$\underbrace{\int \frac{1}{\sqrt{u^2-1}}\,du}_{4} = \underbrace{\cosh^{-1} u}_{4} + \underbrace{C}_{1} = \underbrace{\cosh^{-1}(\cosh^2 x)}_{2} + \underbrace{C}_{1}$

Find $\int_{-1}^{1} \frac{2x^5 - 13x^2}{\sqrt[3]{15 + 13x^3 - x^6}}\,dx$. **SCORE: ___ / 20 POINTS**

$u = 15 + 13x^3 - x^6$ 4 $\quad x = 1 \to u = 27$

$du = (39x^2 - 6x^5)\,dx$ 2 $\quad x = -1 \to u = 1$

$du = 3(13x^2 - 2x^5)\,dx$

$-\frac{1}{3}du = (2x^5 - 13x^2)\,dx$

$\underbrace{-\frac{1}{3}}_{2}\int_{\underbrace{1}_{1}}^{\overset{1}{27}} \underbrace{\frac{1}{\sqrt[3]{u}}\,du}_{2} = -\frac{1}{3}\int_1^{27} u^{-\frac{1}{3}}\,du = -\frac{1}{3}\cdot \underbrace{\frac{3}{2}}_{2}\, \underbrace{u^{\frac{2}{3}}}_{2}\Big|_1^{27} = -\frac{1}{2}\left(27^{\frac{2}{3}} - 1^{\frac{2}{3}}\right)$

$= -\frac{1}{2}(\underbrace{9}_{2} - 1)$

$= \underbrace{-4}_{2}$

Find $\int_1^3 (x^2 - 2x + 1)\,dx$ using the definition of the definite integral and right hand sums. **SCORE: ___ / 20 POINTS**

NO CREDIT IF YOU USE THE FUNDAMENTAL THEOREM OF CALCULUS OR NET CHANGE THEOREM.

$2\ \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n}}_{} \Big(\underbrace{\left(1+\frac{2i}{n}\right)^2 - 2\left(1+\frac{2i}{n}\right) + 1}_{\text{2 FOR SUBSTITUTING}}\Big)\underbrace{\frac{2}{n}}_{}$ 4 $\qquad +2$ FOR $1 + \frac{2i}{n}$

$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \Big(\underbrace{1 + \frac{4i}{n} + \frac{4i^2}{n^2} - 2 - \frac{4i}{n} + 1}_{2}\Big)$

$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \underbrace{\frac{4i^2}{n^2}}_{}$ 2

$= \lim_{n\to\infty} \frac{2}{\cancel{n}} \cdot \underbrace{\frac{4}{n^2}}_{1} \cdot \underbrace{\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}}_{3}$

$= \underbrace{\frac{8}{3}}_{}$ 2