Math 1A (9:30am – 10:20am)
Midterm 1 Version W
Wed Apr 27, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER WORK (SO I CAN TELL HOW YOU GOT YOUR ANSWERS)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem. **SCORE: ___ / 10 POINTS**
Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION B

If f and g are continuous functions such that $f(x) < g(x)$ for all x, and $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$, **SCORE: ___ / 5 POINTS**

what can you conclude about this situation ? **Explain very briefly.**

SEE 7:30 VERSION B

The acceleration of an object at time t (in hours) is given by $a(t) = \sqrt[3]{t}\,\text{sech}\,t$ miles per hour^2, where $t = 0$ corresponds to noon. At three hours before noon, the object's velocity was 11 miles per hour. Find the object's velocity at three hours after noon. **SCORE: ___ / 20 POINTS**

$\underbrace{v(3) - v(-3)}_{4} = \int_{-3}^{3} v'(t)\,dt = \underbrace{\int_{-3}^{3} a(t)\,dt}_{2} = \underbrace{\int_{-3}^{3} \sqrt[3]{t}\,\text{sech}\,t\,dt}_{4}$

$\underbrace{\sqrt[3]{-t}\,\text{sech}(-t)}_{2}$

$= \underbrace{-\sqrt[3]{t}\,\text{sech}\,t}_{2}$

SINCE THE INTEGRAND IS $\underbrace{\text{ODD}}_{2}$, THE INTEGRAL IS 0

$\underbrace{v(3) - v(-3) = 0}_{2} \Rightarrow v(3) = v(-3) = \underbrace{11 \text{ MPH}}_{2}$

Let $s(m) = \int_{6-m^2}^{2m-2} \sqrt{25 - y^2}\, dy$.

SCORE: ___ / 30 POINTS

8 [a] Find $s(1)$.

$$\underbrace{\int_5^0 \sqrt{25-y^2}\,dy}_{2} = \underbrace{-\int_0^5 \sqrt{25-y^2}\,dy}_{1}$$

$$= \underbrace{-\frac{1}{4}(\pi \cdot 5^2)}_{4} = \underbrace{\frac{-25\pi}{4}}_{1}$$

4 [b] Find $s(2)$.

$$\underbrace{\int_2^2 \sqrt{25-y^2}\,dy}_{2} = \underbrace{0}_{2}$$

18 [c] Find $s'(3)$.

$$s'(m) = \frac{d}{dm}\Big[\overbrace{\int_0^{2m-2}\sqrt{25-y^2}\,dy}^{2} \overbrace{-}^{2} \overbrace{\int_0^{6-m^2}\sqrt{25-y^2}\,dy}^{2}\Big]$$

$$= \underbrace{\sqrt{25-(2m-2)^2}}_{2} \cdot \underbrace{2}_{2} - \underbrace{\sqrt{25-(6-m^2)^2}}_{2} \cdot \underbrace{(-2m)}_{2}$$

$$\underbrace{s'(3) = 2\sqrt{9} + 6\sqrt{16}}_{2}$$

$$= 2(3) + 6(4)$$

$$= \underbrace{30}_{2}$$

The graph of $f(x)$ is shown on the right. Let $g(x) = \int_5^x f(t)\, dt$.

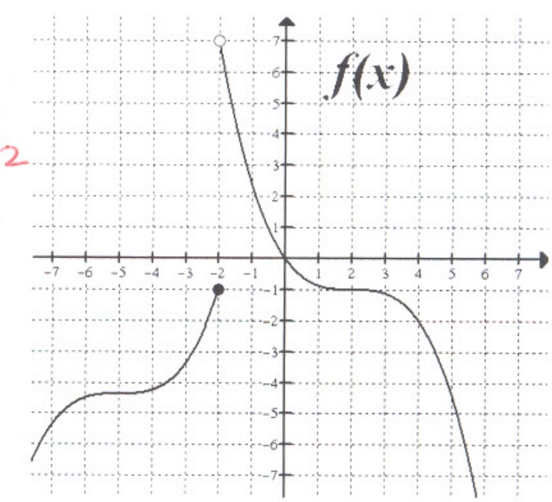
SCORE: ___ / 25 POINTS

6 [a] At what value(s) of x does g have an inflection point ?
Explain very briefly.

$\underbrace{g' = f}_{2}$ CHANGES FROM INCREASING TO DECREASING 2

AT $\underbrace{x = -2}_{2}$

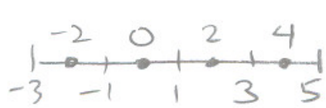
2 (ONLY IF EXPLANATION PROVIDED)



9 [b] Estimate $g(-3)$ using the Midpoint Rule with 4 equal subintervals.

$$\underbrace{g(-3) = \int_5^{-3} f(t)\,dt}_{2}$$

$$= \underbrace{-\int_{-3}^{5} f(t)\,dt}_{1}$$



$$\Delta x = \frac{5-(-3)}{4} = 2$$

$$\approx -\underbrace{(f(-2) + f(0) + f(2) + f(4))}_{8}\, 2$$

$$= -\underbrace{(-1 + 0 + -1 + -2)}_{4}\,\underbrace{2}_{3} = \underbrace{8}_{1}$$

Find $\displaystyle\int \frac{\sinh 2x}{\sqrt{\sinh^4 x+1}}\,dx$.

SCORE: ___ / 20 POINTS

$\underbrace{u=\sinh^2 x}_{4}$

$\underbrace{du = 2\sinh x\cosh x\,dx}_{2}$

$du = \underbrace{\sinh 2x\,dx}_{2}$

$$\underbrace{\int \frac{1}{\sqrt{u^2+1}}\,du}_{4} = \underbrace{\sinh^{-1} u}_{4} + \underbrace{C}_{1} = \underbrace{\sinh^{-1}(\sinh^2 x)}_{2} + \underbrace{C}_{1}$$

Find $\displaystyle\int_{-1}^{1} \frac{2x^4+4x^2}{\sqrt[3]{14+10x^3+3x^5}}\,dx$.

SCORE: ___ / 20 POINTS

$\underbrace{u = 14+10x^3+3x^5}_{4}$

$\underbrace{du = (30x^2+15x^4)\,dx}_{2}$

$du = 15(2x^2+x^4)\,dx$

$\frac{2}{15}du = 2(2x^2+x^4)\,dx$

$= (4x^2+2x^4)\,dx$

$x = 1 \rightarrow u = 27$

$x = -1 \rightarrow u = 1$

$$\underbrace{\frac{2}{15}}_{2}\int_{\underbrace{1}_{1}}^{\overbrace{27}^{1}} \underbrace{\frac{1}{\sqrt[3]{u}}\,du}_{2} = \frac{2}{15}\int_1^{27} u^{-\frac{1}{3}}\,du$$

$$= \frac{\cancel{2}}{\cancel{15}_{5}}\underbrace{\frac{\cancel{3}}{\cancel{2}}}_{2}\underbrace{u^{\frac{2}{3}}}_{2}\Big|_1^{27}$$

$$= \frac{1}{5}\left(27^{\frac{2}{3}} - 1^{\frac{2}{3}}\right)$$

$$= \frac{1}{5}(\underbrace{9-1}_{2}) = \underbrace{\frac{8}{5}}_{2}$$

Find $\displaystyle\int_{2}^{3} (x^2-4x+4)\,dx$ using the definition of the definite integral and right hand sums.

SCORE: ___ / 20 POINTS

NO CREDIT IF YOU USE THE FUNDAMENTAL THEOREM OF CALCULUS OR NET CHANGE THEOREM.

$$\underbrace{\lim_{n\to\infty} \sum_{i=1}^{n}}_{2} \underbrace{\left(\left(2+\frac{i}{n}\right)^2 - 4\left(2+\frac{i}{n}\right)+4\right)}_{\text{2 FOR SUBSTITUTING}} \underbrace{\frac{1}{n}}_{4}$$

+2 FOR $2+\frac{i}{n}$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} \underbrace{\left(4+\frac{4i}{n}+\frac{i^2}{n^2}-8-\frac{4i}{n}+4\right)}_{2}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\underbrace{\frac{i^2}{n^2}}_{2}$$

$$= \lim_{n\to\infty}\frac{1}{\cancel{n}}\cdot\underbrace{\frac{1}{n^2}}_{1}\cdot\underbrace{\frac{\cancel{n}(n+1)(2n+1)}{6}}_{3}$$

$$= \frac{2}{6} = \underbrace{\frac{1}{3}}_{2}$$