NO CALCULATORS ALLOWED

SHOW PROPER WORK (SO I CAN TELL HOW YOU GOT YOUR ANSWERS)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

A 30 foot long chain weighing 40 pounds hangs from the roof of a 30 foot building. You start pulling the chain to the roof. When the bottom of the chain reaches a window 10 feet from the ground, a 25 pound bucket is attached to the chain and pulled to the roof. Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for your work done.

SCORE: ___ / 15 POINTS

$$\int_0^{30} \frac{40}{30} x\,dx + 25 \cdot (30-10)$$

$$= \int_0^{30} \frac{4}{3} x\,dx + 25 \cdot 20 \text{ lb ft}$$

OR $$\int_0^{30} \frac{1}{3}(40-x)\,dx + 25 \cdot 20 \text{ lb ft}$$

The graph of $y = \cosh^{-1} x$ on $[2, 3]$ is revolved around the $y-$axis.

SCORE: ___ / 10 POINTS

Write, **BUT DO NOT EVALUATE**, a dx integral for the area of the resulting surface. **DO NOT SIMPLIFY YOUR ANSWER.**
You may use hyperbolic or inverse hyperbolic notation in your limits of integration if you wish.

$$\int_2^3 2\pi x \sqrt{1 + \left(\frac{1}{\sqrt{x^2-1}}\right)^2}\,dx$$

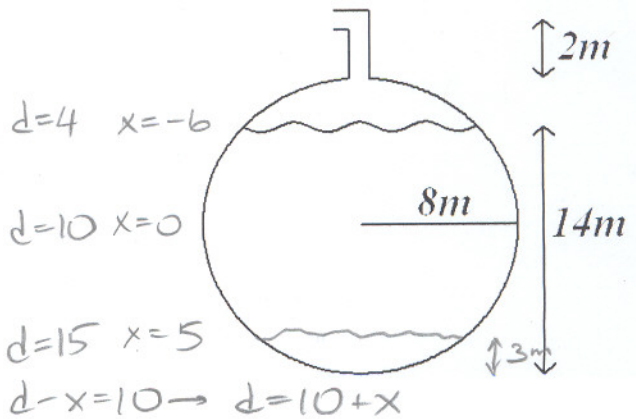
A spherical tank of radius 8 meter has a 2 meter tall spout at the top. The water level in the tank is currently at a height of 14 meters. Write, **BUT DO NOT EVALUATE**, an integral for the work done in pumping the water out through the spout so that the remaining water level in the tank is at a height of 3 meters.

SCORE: ___ / 20 POINTS

$$\int_{-6}^{5} \rho g (10+x)(64-x^2)\,dx \text{ J}$$

$\rho = 1000$, $g = 9.8$

OTHER SOLUTIONS POSSIBLE DEPENDING ON YOUR SCALE



The region bounded by $x = y^2$ and $y = \frac{3-x}{2}$ is revolved around the line $y = 2$.

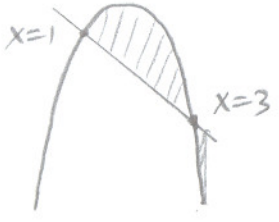
SCORE: ___ / 25 POINTS

Find the volume of the resulting solid.

SEE 7:30 VERSION E

Find the area between the graphs of $f(x) = 12 - 3x^2$ and $g(x) = 21 - 12x$ on the interval $[1, 4]$.

SCORE: ___ / 20 POINTS



$12 - 3x^2 = 21 - 12x$

$0 = 3x^2 - 12x + 9$

$0 = 3(x-1)(x-3)$

$x = 1, 3$

$\int_1^3 (12 - 3x^2 - (21 - 12x))\,dx + \int_3^4 (21 - 12x - (12 - 3x^2))\,dx$

$= \int_1^3 (-9 + 12x - 3x^2)\,dx + \int_3^4 (9 - 12x + 3x^2)\,dx$

$= (-9x + 6x^2 - x^3)\Big|_1^3 + (9x - 6x^2 + x^3)\Big|_3^4$

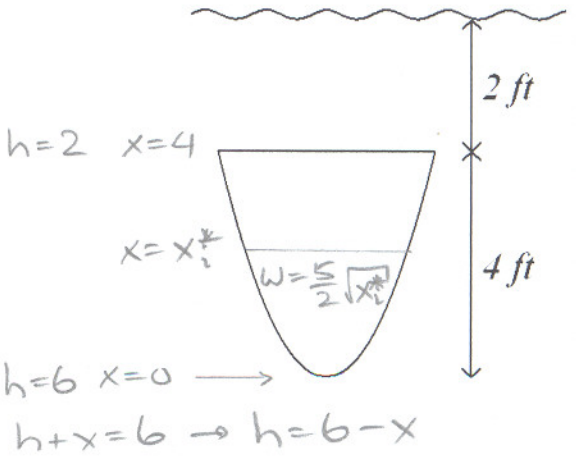
$= -9(3-1) + 6(9-1) - (27-1) + 9(4-3) - 6(16-9) + (64-27)$

$= -18 + 48 - 26 + 9 - 42 + 37$

$= 8$

An aquarium has a 4 foot tall parabolic window with a flat top. The window is 2 feet underwater. **SCORE: ___ / 20 POINTS**

The width of the window x feet above its lowest point is $\frac{5}{2}\sqrt{x}$ feet. Find the hydrostatic force on the window.



$$\int_0^4 \delta(6-x)\tfrac{5}{2}\sqrt{x}\,dx$$

$$= \tfrac{5\delta}{2}\int_0^4 (6x^{\frac{1}{2}} - x^{\frac{3}{2}})\,dx$$

$$= \tfrac{5\delta}{2}\left(4x^{\frac{3}{2}} - \tfrac{2}{5}x^{\frac{5}{2}}\right)\Big|_0^4$$

$$= \tfrac{5\delta}{2}\left(4(8) - \tfrac{2}{5}(32)\right)$$

$$= \delta(80-32)$$

$$= 48\delta \text{ lb ft}$$

$\delta = 62.4$ or 62.5

State the Integral Mean Value Theorem. **SCORE: ___ / 5 POINTS**

Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION E

Find the value of c guaranteed by the Integral Mean Value Theorem for $f(x) = x^2 - 4x - 2$ on the interval $[-3, 3]$. **SCORE: ___ / 20 POINTS**

$$c^2 - 4c - 2 = \frac{1}{3-(-3)}\int_{-3}^{3}(x^2-4x-2)\,dx$$

$$= \tfrac{1}{6}\left(\tfrac{1}{3}x^3 - 2x^2 - 2x\right)\Big|_{-3}^{3}$$

$$= \tfrac{1}{6}\left(\tfrac{1}{3}(27-(-27)) - 2(9-9) - 2(3-(-3))\right)$$

$$c^2 - 4c - 2 = 1$$

$$c^2 - 4c - 3 = 0$$

$$c = \frac{4 \pm \sqrt{16+12}}{2}$$

$$= \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7} \qquad c = 2-\sqrt{7} \in [-3,3]$$

A solid of revolution has volume $\int_1^2 \pi((5+2y)^2-(5-y)^2)\,dy$.

SCORE: ___ / 15 POINTS

Sketch and shade in the region, and draw the axis of revolution.

NOTE: Your axes MUST be in standard position: $y-$ axis up and down, $x-$ axis left and right.

WASHER METHOD

HORIZONTAL CUT → VERTICAL AXIS OF REV

$x=5$

$x=-2y \rightarrow y=-\frac{x}{2}$

$x=y \rightarrow y=x$

FOR $1 \le y \le 2$