NO CALCULATORS ALLOWED

SHOW PROPER WORK (SO I CAN TELL HOW YOU GOT YOUR ANSWERS)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

A 50 foot long rope weighing 60 pounds hangs from the roof of a 50 foot building. You use the rope to pull a 35 pound tabletop from the ground to the roof. 20 feet before it reaches the roof, the tabletop slips loose and crashes to the ground. You pull the rest of the rope to the roof. Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for your work done. **NOTE: Your work includes the lifting, but not the falling, of the tabletop.** **SCORE: ___ / 15 POINTS**

$$\int_0^{50} \frac{60}{50} x\,dx + 35 \cdot (50-20)$$

$$= \int_0^{50} \frac{6}{5} x\,dx + 35 \cdot 30 \text{ lb ft}$$

$$\text{OR } \int_0^{50} \frac{6}{5}(50-x)\,dx + 35 \cdot 30 \text{ lb ft}$$

The graph of $y = \sinh^{-1} x$ on $[2, 3]$ is revolved around the $y-$axis. **SCORE: ___ / 10 POINTS**
Write, **BUT DO NOT EVALUATE**, a dx integral for the area of the resulting surface. **DO NOT SIMPLIFY YOUR ANSWER.**
You may use hyperbolic or inverse hyperbolic notation in your limits of integration if you wish.

$$\int_2^3 2\pi x \sqrt{1 + \left(\frac{1}{\sqrt{x^2+1}}\right)^2}\,dx$$

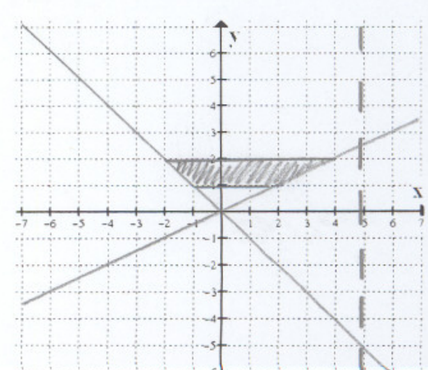
ERRATA

A solid of revolution has volume $\int_1^2 \pi((5-2y)^2 - (5+y)^2)\,dy$. **SCORE: ___ / 15 POINTS**

Sketch and shade in the region, and draw the axis of revolution.
NOTE: Your axes MUST be in standard position: $y-$ **axis up and down,** $x-$ **axis left and right.**

WASHER METHOD
HORIZONTAL CUT → VERTICAL AXIS OF REV
$x=5$

$x = 2y \rightarrow y = \frac{x}{2}$
$x = y \rightarrow y = x$ FOR $1 \le y \le 2$



State the Integral Mean Value Theorem. **SCORE: ___ / 5 POINTS**

Use complete sentences and proper algebra & English as shown in class.

IF f IS CONTINUOUS ON $[a, b]$
THEN THERE EXISTS $c \in [a, b]$
SUCH THAT $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

Find the value of c guaranteed by the Integral Mean Value Theorem for $f(x) = x^2 - 4x + 3$ on the interval $[-3, 3]$. **SCORE: ___ / 20 POINTS**

$$c^2 - 4c + 3 = \frac{1}{3 - -3}\int_{-3}^{3}(x^2 - 4x + 3)\,dx$$

$$= \frac{1}{6}\left(\frac{1}{3}x^3 - 2x^2 + 3x\right)\Big|_{-3}^{3}$$

$$= \frac{1}{6}\left(\frac{1}{3}(27 - -27) - 2(9 - 9) + 3(3 - -3)\right)$$

$$c^2 - 4c + 3 = 6$$

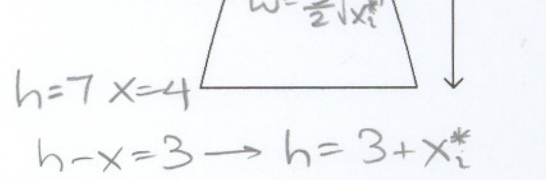
$$c^2 - 4c - 3 = 0$$

$$c = \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7} \qquad c = 2 - \sqrt{7} \in [-3, 3]$$

An aquarium has a 4 foot tall parabolic window with a flat bottom. The window is 3 feet underwater. **SCORE: ___ / 20 POINTS**

The width of the window x feet below its highest point is $\frac{5}{2}\sqrt{x}$ feet. Find the hydrostatic force on the window.



$$\int_0^4 \delta(3 + x)\frac{5}{2}\sqrt{x}\,dx$$

$$= \frac{5\delta}{2}\int_0^4 (3x^{\frac{1}{2}} + x^{\frac{3}{2}})\,dx$$

$$= \frac{5\delta}{2}\left(2x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}}\right)\Big|_0^4$$

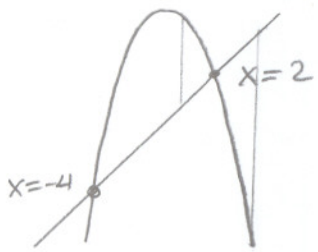
$$= \frac{5\delta}{2}\left(2(8) + \frac{2}{5}(32)\right)$$

$$= \delta(40 + 32)$$

$$= 72\delta \text{ lb ft}$$

$$\delta = 62.4 \text{ or } 62.5$$

Find the area between the graphs of $f(x) = 27 - 3x^2$ and $g(x) = 3 + 6x$ on the interval $[1, 4]$. **SCORE: ___ / 20 POINTS**

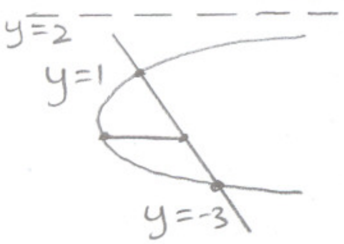


$$27 - 3x^2 = 3 + 6x$$
$$0 = 3x^2 + 6x - 24$$
$$0 = 3(x+4)(x-2)$$
$$x = -4, 2$$

$$\int_1^2 (27 - 3x^2 - (3+6x))\,dx + \int_2^4 (3 + 6x - (27 - 3x^2))\,dx$$
$$= \int_1^2 (24 - 6x - 3x^2)\,dx + \int_2^4 (3x^2 + 6x - 24)\,dx$$
$$= (24x - 3x^2 - x^3)\Big|_1^2 + (x^3 + 3x^2 - 24x)\Big|_2^4$$
$$= 24(2-1) - 3(4-1) - (8-1) + (64-8) + 3(16-4) - 24(4-2)$$
$$= 24 - 9 - 7 + 56 + 36 - 48$$
$$= 52$$

The region bounded by $x = y^2$ and $y = \dfrac{3-x}{2}$ is revolved around the line $y = 2$. **SCORE: ___ / 25 POINTS**

Find the volume of the resulting solid.



$$y^2 = 3 - 2y$$
$$y^2 + 2y - 3 = 0$$
$$(y+3)(y-1) = 0$$
$$y = -3, 1$$

$$\int_{-3}^{1} 2\pi (2-y)(3 - 2y - y^2)\,dy$$
$$= 2\pi \int_{-3}^{1} (6 - 4y - 2y^2 - 3y + 2y^2 + y^3)\,dy$$
$$= 2\pi \int_{-3}^{1} (6 - 7y + y^3)\,dy$$
$$= 2\pi \left(6y - \tfrac{7}{2}y^2 + \tfrac{1}{4}y^4\right)\Big|_{-3}^{1}$$
$$= 2\pi \left(6(1 - -3) - \tfrac{7}{2}(1-9) + \tfrac{1}{4}(1-81)\right)$$
$$= 2\pi(24 + 28 - 20) = 64\pi$$

A spherical tank of radius 7 meter has a 1 meter tall spout at the top. The water level in the tank is currently at a height of 12 meters. Write, **BUT DO NOT EVALUATE**, an integral for the work done in pumping the water out through the spout so that the remaining water level in the tank is at a height of 3 meters.

SCORE: ___ / 20 POINTS

$$\int_{-5}^{4} \rho g (8+x)(49-x^2)\, dx \text{ J}$$

$$\rho = 1000, \; g = 9.8$$

OTHER SOLUTIONS POSSIBLE DEPENDING ON YOUR SCLAE

$d=3 \quad x=-5$

$d=8 \quad x=0$

$d=12 \quad x=4$

$d-x=8 \rightarrow d=8+x$