NO CALCULATORS ALLOWED

SHOW PROPER WORK (SO I CAN TELL HOW YOU GOT YOUR ANSWERS)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

A 30 foot long chain weighing 40 pounds hangs from the roof of a 30 foot building. You start pulling the chain to the roof. When the bottom of the chain reaches a window 10 feet from the ground, a 25 pound bucket is attached to the chain and pulled to the roof. Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for your work done. **SCORE: ___ / 15 POINTS**

SEE 7:30 VERSION I

The graph of $y = \cosh^{-1} x$ on $[2, 3]$ is revolved around the $y-$axis. Write, **BUT DO NOT EVALUATE**, a dx integral for the area of the resulting surface. **DO NOT SIMPLIFY YOUR ANSWER.** **You may use hyperbolic or inverse hyperbolic notation in your limits of integration if you wish.** **SCORE: ___ / 10 POINTS**

SEE 7:30 VERSION I

A spherical tank of radius 8 meter has a 2 meter tall spout at the top. The water level in the tank is currently at a height of 14 meters. Write, **BUT DO NOT EVALUATE**, an integral for the work done in pumping the water out through the spout so that the remaining water level in the tank is at a height of 3 meters. **SCORE: ___ / 20 POINTS**

SEE 7:30 VERSION I
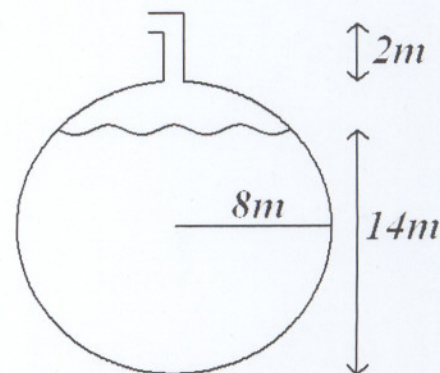
The region bounded by $x = y^2$ and $y = \dfrac{3-x}{2}$ is revolved around the line $y = 2$. Find the volume of the resulting solid.

SCORE: ___ / 25 POINTS

SEE 7:30 VERSION E

Find the area between the graphs of $f(x) = 12 - 3x^2$ and $g(x) = 3 - 6x$ on the interval $[1, 4]$.

SCORE: ___ / 20 POINTS

$12 - 3x^2 = 3 - 6x$

$0 = 3x^2 - 6x - 9$

$0 = 3(x - 3)(x + 1)$

$x = -1, 3$

$\displaystyle\int_1^3 (12 - 3x^2 - (3 - 6x))\, dx + \int_3^4 (3 - 6x - (12 - 3x^2))\, dx$

$= \displaystyle\int_1^3 (9 + 6x - 3x^2)\, dx + \int_3^4 (3x^2 - 6x - 9)\, dx$

$= (9x + 3x^2 - x^3)\big|_1^3 + (x^3 - 3x^2 - 9x)\big|_3^4$

$= 9(3-1) + 3(9-1) - (27-1) + (64-27) - 3(16-9) - 9(4-3)$
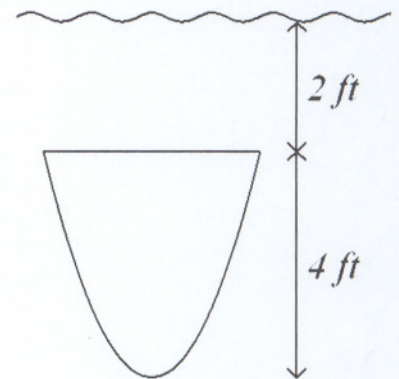
$= 18 + 24 - 26 + 37 - 21 - 9$

$= 23$

An aquarium has a 4 foot tall parabolic window with a flat top. The window is 2 feet underwater. **SCORE: ___ / 20 POINTS**

The width of the window x feet above its lowest point is $\dfrac{5}{2}\sqrt{x}$ feet. Find the hydrostatic force on the window.

SEE 7:30 VERSION I

2 ft

4 ft

State the Integral Mean Value Theorem. **SCORE: ___ / 5 POINTS**
Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION E

Find the value of c guaranteed by the Integral Mean Value Theorem **SCORE: ___ / 20 POINTS**
for $f(x) = x^2 - 4x - 3$ on the interval $[-3, 3]$.

$$c^2 - 4c - 3 = \frac{1}{3 - {}^-3} \int_{-3}^{3} (x^2 - 4x - 3)\, dx$$

$$= \frac{1}{6} \left(\frac{1}{3}x^3 - 2x^2 - 3x \right) \Big|_{-3}^{3}$$

$$= \frac{1}{6} \left(\frac{1}{3}(27 - {}^-27) - 2(9 - 9) - 3(3 - {}^-3) \right)$$

$$c^2 - 4c - 3 = 0$$

$$c = \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7} \qquad c = 2 - \sqrt{7} \in [-3, 3]$$

A solid of revolution has volume $\int_{1}^{2} \pi((5+2y)^2 - (5-y)^2)\, dy$.

Sketch and shade in the region, and draw the axis of revolution.

NOTE: Your axes MUST be in standard position: $y-$ **axis up and down,** $x-$ **axis left and right.**

SEE 7:30 VERSION I