Math 1A (7:30am – 8:20am)
Quiz 1 Version K
Mon Apr 11, 2011

NAMES YOU ASKED TO BE CALLED IN CLASS:

_________________________ & _________________________

SCORE: ___ / 20 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (INCLUDING ALL IDENTITIES USED)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\lim\limits_{x\to 0^-} \coth x$. **Do NOT use a graph. Give algebraic or numerical reasoning, as shown in class.** SCORE: ___ / 2 POINTS

$$\lim_{x\to 0^-} \frac{\cosh x}{\sinh x} = \underbrace{-\infty}_{1} \quad \text{OR} \quad \lim_{x\to 0^-} \frac{e^x+e^{-x}}{e^x-e^{-x}} = \lim_{x\to 0^-} \frac{e^{2x}+1}{e^{2x}-1} = -\infty$$

$$\underbrace{\left(\frac{1}{0^-}\right)}_{1} \quad \text{OR} \quad \left(\frac{2}{1^- - 1^+} \to \frac{2}{0^-}\right) \quad \text{OR} \quad \left(\frac{2}{1^- - 1} \to \frac{2}{0^-}\right)$$

State the definition of "area under a function" given in class.
Use complete sentences and proper algebra & English as shown in class. SCORE: ___ / 2 POINTS

IF f IS CONTINUOUS + NON-NEGATIVE ON $[a,b]$
THEN THE AREA UNDER f ON $[a,b]$ IS
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\Delta x \text{ WHERE } \Delta x = \frac{b-a}{n}$$

Using the definition of "area under a function" given in class, write an algebraic expression for the area under $f(x) = \sin 2x$ over the interval $[3, 7]$. **Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.** SCORE: ___ / 2 POINTS

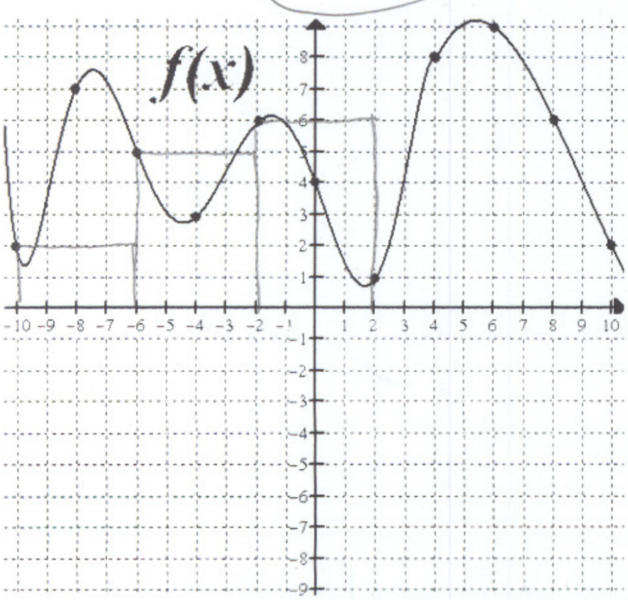
$$\underbrace{\lim_{n\to\infty}}_{\frac{1}{4}} \underbrace{\sum_{i=1}^{n}}_{\frac{1}{4}} \underbrace{\sin 2(3+\underbrace{\tfrac{4i}{n}}_{\frac{1}{2}}) \cdot \underbrace{\tfrac{4}{n}}_{\frac{1}{2}}}_{\frac{1}{2}}$$

OR $\sum\limits_{i=0}^{n-1}$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER SCORE: ___ / 2 POINTS

For the function f on the interval $[-10, 2]$, A_3 using the left hand sum (known as L_3 in your textbook) equals

[a] 48 [b] 52 [c] 56 [d] 60 [e] 64 [f] none of the above



$$\Delta x = \frac{2-(-10)}{3} = 4$$

$$4(2+5+6) = 52$$

Find $\frac{d}{dx}\tanh^{-1}(\operatorname{sech} x)$. SCORE: ___ / 3 POINTS

$$\underbrace{\frac{1}{1-\operatorname{sech}^2 x}}_{1} \cdot \underbrace{-\operatorname{sech} x \tanh x}_{1} = \underbrace{\frac{1}{\tanh^2 x}}_{\frac{1}{2}} - \operatorname{sech} x \tanh x$$

$$= \underbrace{\frac{-\operatorname{sech} x}{\tanh x}}_{\frac{1}{4}} \cdot \frac{\cosh x}{\cosh x} = \frac{-1}{\sinh x} = \underbrace{-\operatorname{csch} x}_{\frac{1}{4}}$$

If $\sinh x = -3$, find $\cosh 2x$, **using identities**. SCORE: ___ / 3 POINTS
Do NOT use the logarithmic formula for any inverse hyperbolic functions.

$$\cosh 2x = \underbrace{2\sinh^2 x + 1}_{2}$$
$$= 2(-3)^2 + 1$$
$$= \underbrace{19}_{1}$$

OR

OR

$$\underbrace{\cosh^2 x - \sinh^2 x = 1}_{\frac{1}{2}}$$
$$\cosh^2 x - (-3)^2 = 1$$
$$\underbrace{\cosh^2 x = 10}_{\frac{1}{2}} \Rightarrow \cosh x = \sqrt{10}$$
$$\cosh 2x = \underbrace{\cosh^2 x + \sinh^2 x}_{} = \underbrace{19}_{1}$$
$$\text{OR} = \underbrace{2\cosh^2 x - 1}_{1} = 19$$

OR

Prove the logarithmic formula for $\tanh^{-1} x$. SCORE: ___ / 3 POINTS

$$y = \tanh^{-1} x$$
$$\tanh y = x$$
$$\boxed{\frac{e^{2y}-1}{e^{2y}+1} = x}\ 1 \quad \text{LET } z = e^{2y}$$
$$\boxed{\frac{z-1}{z+1} = x}\ \frac{1}{4}$$
$$\underbrace{z - 1 = xz + x}_{\frac{1}{4}}$$
$$z - xz = 1 + x$$
$$\underbrace{z(1-x) = 1 + x}_{\frac{1}{4}}$$

$$z = \frac{1+x}{1-x}$$
$$\underbrace{e^{2y} = \frac{1+x}{1-x}}_{\frac{1}{2}}$$
$$\underbrace{2y = \ln\frac{1+x}{1-x}}_{\frac{1}{4}}$$
$$y = \underbrace{\frac{1}{2}\ln\frac{1+x}{1-x}}_{\frac{1}{2}}$$

Prove the derivative of $\sinh^{-1} x$. **Do NOT use the logarithmic formula for** $\sinh^{-1} x$. SCORE: ___ / 3 POINTS

$$y = \sinh^{-1} x$$
$$\underbrace{\sinh y = x}_{\frac{1}{2}}$$
$$\frac{d}{dx}\sinh y = \frac{d}{dx}x$$
$$\underbrace{\cosh y \frac{dy}{dx} = 1}_{1}$$
$$\underbrace{\frac{dy}{dx} = \frac{1}{\cosh y}}_{\frac{1}{2}}$$
$$= \boxed{\frac{1}{\sqrt{1+\sinh^2 y}}}\ \frac{1}{2} = \underbrace{\frac{1}{\sqrt{1+x^2}}}_{\frac{1}{2}}$$

$$\cosh^2 y - \sinh^2 y = 1$$
$$\cosh^2 y = 1 + \sinh^2 y$$
$$\cosh y = \sqrt{1+\sinh^2 y}$$
SINCE $\cosh y > 0$
FOR ALL x