Math 1A (7:30am – 8:20am)
Quiz 1 Version R
Mon Apr 11, 2011

NAMES YOU ASKED TO BE CALLED IN CLASS:

________________________ & ________________________

SCORE: ___ / 20 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (INCLUDING ALL IDENTITIES USED)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\lim_{x\to 0^+} \coth x$. **Do NOT use a graph. Give algebraic or numerical reasoning, as shown in class.** SCORE: ___ / 2 POINTS

$\lim_{x\to 0^+}\frac{\cosh x}{\sinh x} = \infty$, $\left(\frac{1}{0^+}\right)$ OR $\lim_{x\to 0^+}\frac{e^x+e^{-x}}{e^x-e^{-x}} = \lim_{x\to 0^+}\frac{e^{2x}+1}{e^{2x}-1} = \infty$

$\left(\frac{2}{1^+-1^-}\to\frac{2}{0^+}\right)$ $\left(\frac{2}{1^+-1}\to\frac{2}{0^+}\right)$

OR OR

State the definition of "area under a function" given in class. SCORE: ___ / 2 POINTS
Use complete sentences and proper algebra & English as shown in class.

IF f IS CONTINUOUS + NON-NEGATIVE ON $[a,b]$
THEN THE AREA UNDER f ON $[a,b]$ IS
$\lim_{n\to\infty}\sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x$ WHERE $\Delta x = \frac{b-a}{n}$

Using the definition of "area under a function" given in class, write an algebraic expression for the area under $f(x)=\cos 2x$ over the interval $[4, 9]$. **Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.** SCORE: ___ / 2 POINTS

$\lim_{n\to\infty}\sum_{i=1}^{n}\cos 2\left(4+\frac{5i}{n}\right)\cdot\frac{5}{n}$

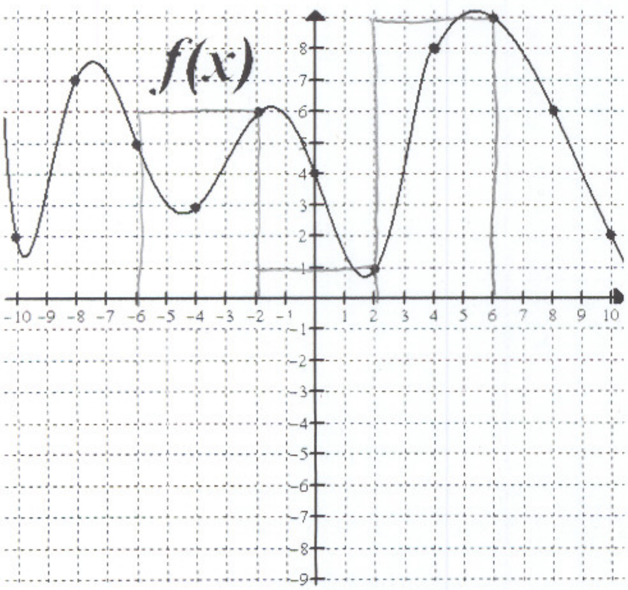
$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

OR $\sum_{i=0}^{n-1}$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER SCORE: ___ / 2 POINTS

For the function f on the interval $[-6, 6]$, A_3 using the right hand sum (known as R_3 in your textbook) equals

[a] 48 [b] 52 [c] 56 [d] 60 [e] 64 [f] none of the above



$\Delta x = \frac{6-(-6)}{3} = 4$

$4(6+1+9) = 64$

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{csch} x)$. **SCORE: ___ / 3 POINTS**

$$\underbrace{\frac{1}{\sqrt{1+\operatorname{csch}^2 x}}}_{1} \cdot \underbrace{-\operatorname{csch} x \coth x}_{1} = \frac{1}{\underbrace{\sqrt{\coth^2 x}}_{\frac{1}{2}}} \cdot -\operatorname{csch} x \coth x$$

$$= \frac{1}{\underbrace{|\coth x|}_{\frac{1}{4}}} \cdot -\operatorname{csch} x \coth x = \underbrace{\pm \operatorname{csch} x}_{\frac{1}{4}}$$

ACTUAL ANSWER IS $-|\operatorname{csch} x|$

If $\sinh x = -5$, find $\cosh 2x$, **using identities**. **SCORE: ___ / 3 POINTS**
Do NOT use the logarithmic formula for any inverse hyperbolic functions.

$$\cosh 2x = \underbrace{2\sinh^2 x + 1}_{2}$$
$$= 2(-5)^2 + 1$$
$$= \underbrace{51}_{1}$$

OR

$$\underbrace{\cosh^2 x - \sinh^2 x = 1}_{\frac{1}{2}}$$
$$\cosh^2 x - (-5)^2 = 1$$
$$\underbrace{\cosh^2 x = 26}_{\frac{1}{2}} \Rightarrow \cosh x = \sqrt{26}$$

OR

$$\cosh 2x = \underbrace{\cosh^2 x + \sinh^2 x}_{} = \underbrace{51}_{1}$$

OR $= \underbrace{2\cosh^2 x - 1}_{1} = 51$

OR

Prove the logarithmic formula for $\tanh^{-1} x$. **SCORE: ___ / 3 POINTS**

$$y = \tanh^{-1} x$$
$$\tanh y = x$$
$$\underbrace{\boxed{\frac{e^y - e^{-y}}{e^y + e^{-y}} = x}}_{1}$$

LET $z = e^y$

$$\underbrace{\boxed{\frac{z - \frac{1}{z}}{z + \frac{1}{z}} = x}}_{\frac{1}{4}}$$
$$\frac{z^2 - 1}{z^2 + 1} = x$$
$$\underbrace{z^2 - 1 = xz^2 + x}_{\frac{1}{4}}$$
$$z^2 - xz^2 = 1 + x$$
$$\underbrace{z^2(1-x) = 1 + x}_{\frac{1}{4}}$$
$$z^2 = \frac{1+x}{1-x}$$
$$\underbrace{e^{2y} = \frac{1+x}{1-x}}_{\frac{1}{2}}$$
$$\underbrace{2y = \ln\frac{1+x}{1-x}}_{\frac{1}{4}}$$
$$y = \underbrace{\frac{1}{2}\ln\frac{1+x}{1-x}}_{\frac{1}{2}}$$

Prove the derivative of $\tanh^{-1} x$. **Do NOT use the logarithmic formula for** $\tanh^{-1} x$. **SCORE: ___ / 3 POINTS**

$$y = \tanh^{-1} x$$
$$\underbrace{\tanh y = x}_{\frac{1}{2}}$$
$$\frac{d}{dx}\tanh y = \frac{d}{dx}x$$
$$\underbrace{\operatorname{sech}^2 y \frac{dy}{dx} = 1}_{1}$$
$$\frac{dy}{dx} = \underbrace{\frac{1}{\operatorname{sech}^2 y}}_{\frac{1}{2}} = \underbrace{\frac{1}{1-\tanh^2 y}}_{\frac{1}{2}} = \underbrace{\frac{1}{1-x^2}}_{\frac{1}{2}}$$