← NO SCORE YET

SCORE: ___ / 20 POINTS

**NO CALCULATORS ALLOWED**

**SHOW PROPER ALGEBRAIC WORK (INCLUDING ALL IDENTITIES USED)**
**USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE**

Find $\lim_{x\to 0^-} \coth x$. **Do NOT use a graph. Give algebraic or numerical reasoning, as shown in class.** SCORE: 2 / 2 POINTS

$\lim_{x\to 0^-} \coth x$

$= \lim_{x\to 0^-} \frac{e^{2x}+1}{e^{2x}-1} = \boxed{-\infty}$ 1

$\frac{1+1}{1^- -1} = \frac{2}{0^-}$ 1

State the definition of "area under a function" given in class. SCORE: 2 / 2 POINTS
**Use complete sentences and proper algebra & English as shown in class.**

If $f$ is continuous and non-negative on $[a,b]$, then the area under $f$ on $[a,b]$ is

$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{k=1}^{n} f(a+k\Delta x)\Delta x$, where $\Delta x = \frac{b-a}{n}$

Using the definition of "area under a function" given in class, write an algebraic expression for the area under SCORE: 2 / 2 POINTS
$f(x) = \cos 3x$ over the interval $[5, 11]$. **Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.**

f is cont. and is non-neg. $\Delta x = \frac{11-5}{n} = \frac{6}{n}$

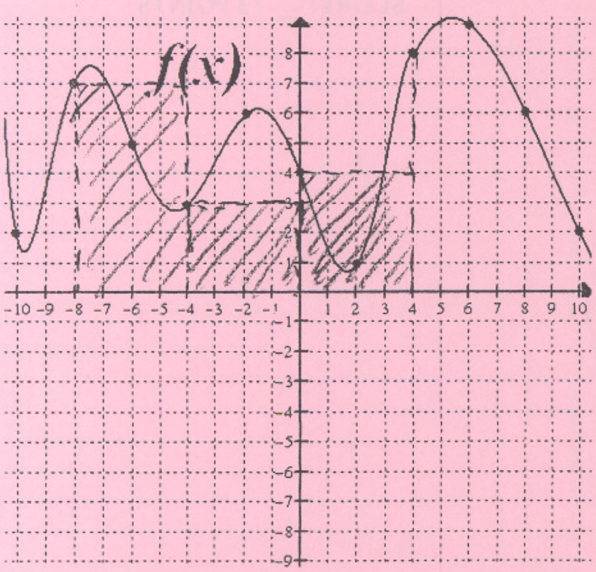
$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(5+k\frac{6}{n}\right)\cdot\frac{6}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \cos 3\left(5+k\frac{6}{n}\right)\cdot\frac{6}{n}$ $\frac{1}{2}$ $\frac{1}{2}$ * BLO

$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{6}{n}\cos\left(15+\frac{18k}{n}\right) = \underbrace{\lim_{n\to\infty} \frac{6}{n}}_{\frac{1}{4}} \underbrace{\sum_{k=1}^{n}}_{\frac{1}{4}} \underbrace{\cos\left(15+\frac{18k}{n}\right)}_{\frac{1}{2}}$

**MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER** SCORE: 2 / 2 POINTS

For the function $f$ on the interval $[-8, 4]$, $A_3$ using the left hand sum (known as $L_3$ in your textbook) equals

[a] 48 [b] 52 [c] 56 [d] 60 [e] 64 [f] none of the above




$4(7+3+4)$

$4(14)$

$56$

I SIGNED OFF WHEN THEY CAME TO MY OFFICE HOURS TO DEFEND THEIR WORK.

HOWEVER, THIS DIFFERENCE IS SO MINOR + OBVIOUSLY ALGEBRAICALLY EQUIVALENT, THEY DIDN'T NEED TO MARK IT WITH (*) OR TO SEE ME ind $\frac{d}{dx}\sinh^{-1}(\text{csch}\, x)$.

$\coth^2 x - 1 = \text{csch}^2 x$

SCORE: ___ / 3 POINTS

NO SCORE YET

$$\frac{1}{\sqrt{1+(\text{csch}x)^2}} \cdot -\text{csch}x\coth x = \frac{-\text{csch}x\coth x}{\sqrt{\coth^2 x}} \quad \frac{1}{2}$$

1 $$= \frac{-\text{csch}x\coth x}{\sqrt{1+\text{csch}^2 x}} = \frac{-\text{csch}x\coth x}{\coth x} \quad *$$

$$= \boxed{-\text{csch}x} \quad *$$

THEY WILL COME TO MY OFFICE HOURS TO DEFEND THIS

If $\sinh x = -6$, find $\cosh 2x$, **using identities**.
Do NOT use the logarithmic formula for any inverse hyperbolic functions.

SCORE: 3 / 3 POINTS

$\cosh 2x = \cosh^2 x + \sinh^2 x$

Identity $\cosh^2 x - \sinh^2 x = 1$ $\frac{1}{2}$

$\cosh^2 x - 36 = 1$

$\cosh^2 x = 37$ $\frac{1}{2}$

$\sinh^2 x = 36$

$\cosh 2x = \cosh^2 x + \sinh^2 x$ 1

$= 37 + 36$

$= \boxed{73}$ 1

Prove the logarithmic formula for $\tanh^{-1} x$.

SCORE: 3 / 3 POINTS

$\frac{1}{2}\ln\left(\frac{x+1}{1-x}\right)$

let $y = \tanh^{-1} x$

$\tanh y = x$

$\frac{e^{2y}-1}{e^{2y}+1} = x$ 1

let $z = e^{2y}$

$\frac{z-1}{z+1} = x$ $\frac{1}{4}$

$z - 1 = xz + x$ $\frac{1}{4}$

$z - xz = x + 1$

$z(1-x) = 1+x$ $\frac{1}{4}$

$z = \frac{1+x}{1-x}$

$z = e^{2y}$

$e^{2y} = \frac{1+x}{1-x}$ $\frac{1}{2}$

$\ln(e^{2y}) = \ln\left(\frac{1+x}{1-x}\right)$

$2y = \ln\left(\frac{1+x}{1-x}\right)$ $\frac{1}{4}$

$y = \frac{1}{2}\left(\ln\left(\frac{1+x}{1-x}\right)\right)$ $\frac{1}{2}$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

Prove the derivative of $\tanh^{-1} x$. **Do NOT use the logarithmic formula for** $\tanh^{-1} x$.

SCORE: 3 / 3 POINTS

let $y = \tanh^{-1} x$

$\tanh y = x$ $\frac{1}{2}$

$\frac{d}{dx}\tanh y = \frac{d}{dx} x$

$\text{sech}^2 y \frac{dy}{dx} = 1$ 1

$\frac{dy}{dx} = 1/\text{sech}^2 y$ $\frac{1}{2}$

$\cosh^2 y - \sinh^2 y = 1$

$1 - \tanh^2 y = \text{sech}^2 y$

$\frac{dy}{dx} = 1/(1-\tanh^2 y)$ $\frac{1}{2}$

$= 1/(1-x^2)$ $\frac{1}{2}$

$$\frac{d}{dx}\tanh^{-1} x = 1/(1-x^2)$$