Math 1A (7:30am – 8:20am)
Quiz 2 Version D
Fri Apr 15, 2011

NAMES YOU ASKED TO BE CALLED IN CLASS:

____________________ & ____________________

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (DO NOT USE ANTI-DERIVATIVES)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State the definition of "definite integral" given in class. SCORE: ___ / 3 POINTS
Use complete sentences and proper algebra & English as shown in class.

THE DEFINITE INTEGRAL OF f ON $[a,b]$ IS
$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ WHERE $\Delta x = \frac{b-a}{n}$ AND $a+(i-1)\Delta x \le x_i^* \le a+i\Delta x$
IF THE LIMIT EXISTS AND IS THE SAME REGARDLESS OF THE CHOICE OF x_i^*

Evaluate $\int_{-2}^{1} (2x^2-1)\,dx$ using the definition of the definite integral and right hand sums. SCORE: ___ / 8 POINTS

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2\left(-2+\frac{3i}{n}\right)^2-1\right)\frac{3}{n} \quad 2$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(2\left(4-\frac{12i}{n}+\frac{9i^2}{n^2}\right)-1\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(7-\frac{24i}{n}+\frac{18i^2}{n^2}\right) \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{n} \left[\sum_{i=1}^{n} 7 - \frac{24}{n}\sum_{i=1}^{n} i + \frac{18}{n^2}\sum_{i=1}^{n} i^2\right] \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{\cancel{n}} \left[7\cancel{n} - \frac{\overset{12}{\cancel{24}}}{n}\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\overset{3}{\cancel{18}}}{n^2}\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right] \quad 1 \quad 1 \quad 1$$

$$= \lim_{n\to\infty} 3\left[7-\frac{12(n+1)}{n}+\frac{3(n+1)(2n+1)}{n^2}\right]$$

$$= 3[7-12+6] \quad 1$$

$$= 3 \quad 1$$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER SCORE: ___ / 3 POINTS

$\int_{-3}^{7} f(x)\,dx =$

[a] -17

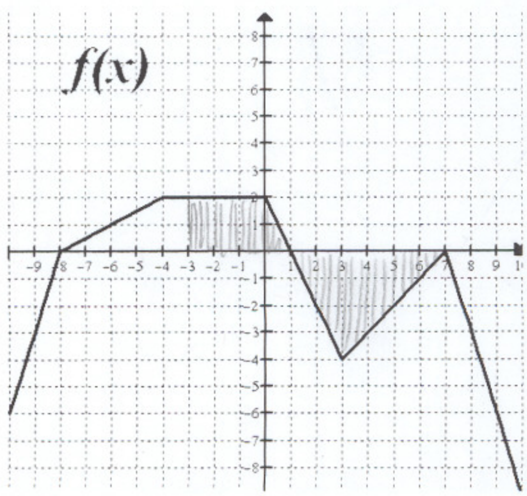
[b] -5

[c] 2

[d] 5

[e] 9

[f] none of the above



$\frac{1}{2}(3+4)2$
$-\frac{1}{2}(4)(6)$
$= -5$

If the velocity of a particle at time t is given by $v(t) = t^2 - 1$, estimate the distance travelled by the object from $t = 1$ to $t = 7$ using 3 equal subintervals and the midpoint rule. **SCORE: ___ / 3 POINTS**

2 4 6

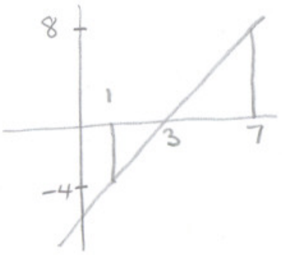
1 3 5 7

$\Delta x = \frac{7-1}{3} = 2$

$$\underbrace{(2^2-1)2 + (4^2-1)2 + (6^2-1)2}_{} = \underbrace{3\cdot 2 + 15\cdot 2 + 35\cdot 2}_{2} = \underbrace{106}_{1}$$

OR

Evaluate $\int_1^7 (2x-6)\,dx$ using geometry. **Sketch a diagram and show the calculations used.** **SCORE: ___ / 3 POINTS**



$$\underbrace{-\tfrac{1}{2}(4)(3-1) + \tfrac{1}{2}(8)(7-3)}_{} = \underbrace{-\tfrac{1}{2}(4)(2) + \tfrac{1}{2}(8)(4)}_{2}$$

OR

$= -4 + 16$

$= \underbrace{12}_{1}$

Write $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{5}{n}\sqrt{3 + \frac{5k}{n}}$ as a definite integral. **Do NOT evaluate the limit/definite integral.** **SCORE: ___ / 3 POINTS**

$$\underbrace{\int}_{\frac{1}{4}}{}_{\underbrace{3}_{\frac{1}{2}}}^{\overbrace{8}^{1}} \underbrace{\sqrt{x}}_{1}\, \underbrace{dx}_{\frac{1}{4}}$$

Write $\int_{-2}^{-4} g(x)\,dx - \int_{4}^{2} g(x)\,dx + \int_{-4}^{2} g(x)\,dx$ as a single integral in the form $\int_a^b g(x)\,dx$. **SCORE: ___ / 3 POINTS**

Show each step CLEARLY.

$$\underbrace{\int_{-2}^{-4} g(x)\,dx + \int_{2}^{4} g(x)\,dx + \int_{-4}^{2} g(x)\,dx}_{}\ 1$$

$$= \underbrace{\int_{-2}^{-4} g(x)\,dx + \int_{-4}^{4} g(x)\,dx}_{} \quad \text{OR} \quad \underbrace{\int_{-2}^{2} g(x)\,dx + \int_{2}^{4} g(x)\,dx}_{}\ 1$$

OR

$$= \underbrace{\int_{-2}^{4} g(x)\,dx}_{}\ 1$$

If $\int_3^8 g(x)\,dx = -7$, find $\int_8^3 (2 + g(x))\,dx$. **Show each step CLEARLY.** **SCORE: ___ / 4 POINTS**

$$= \underbrace{-\int_3^8 (2 + g(x))\,dx}_{1}$$

$$= -\underbrace{\left[\int_3^8 2\,dx + \int_3^8 g(x)\,dx\right]}_{1}$$

$$= -\left[\underbrace{2(8-3) + (-7)}_{1}\right] = \underbrace{-[10-7]}_{} = \underbrace{-3}_{1}$$

OR

OR

$$\underbrace{= \int_8^3 2\,dx + \int_8^3 g(x)\,dx}_{1}$$

$$= \underbrace{2(3-8)}_{1} - \underbrace{\int_3^8 g(x)\,dx}_{1}$$

$$= \underbrace{-10-(-7)}_{1} = \underbrace{-3}_{1}$$