

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

**SHOW PROPER ALGEBRAIC WORK (DO NOT USE ANTI-DERIVATIVES)
 USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE**

State the definition of "definite integral" given in class.

SCORE: ___ / 3 POINTS

Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION D

Evaluate $\int_{-1}^2 (4x^2 - 1) dx$ using the definition of the definite integral and right hand sums.

SCORE: ___ / 8 POINTS

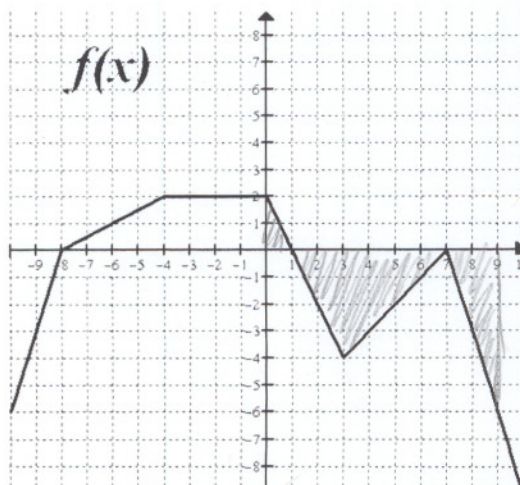
$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \sum_{i=1}^n (4(-1 + \frac{3i}{n})^2 - 1) \frac{3}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n (4(1 - \frac{6i}{n} + \frac{9i^2}{n^2}) - 1) \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n (3 - \frac{24i}{n} + \frac{36i^2}{n^2}) \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n 3 - \frac{24}{n} \sum_{i=1}^n i + \frac{36}{n^2} \sum_{i=1}^n i^2 \right] \\
 &= \lim_{n \rightarrow \infty} \frac{3}{n} \left[3n - \frac{24}{n} \frac{n(n+1)}{2} + \frac{36}{n^2} \frac{n(n+1)(2n+1)}{6} \right] \\
 &= \lim_{n \rightarrow \infty} 3 \left[3 - \frac{12(n+1)}{n} + \frac{6(n+1)(2n+1)}{n^2} \right] \\
 &= 3 [3 - 12 + 12] \\
 &= 9
 \end{aligned}$$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER

SCORE: ___ / 3 POINTS

$$\int_0^9 f(x) dx =$$

- [a] -17
- [b] -5
- [c] 2
- [d] 5
- [e] 9
- [f] none of the above



$$\begin{aligned}
 & \frac{1}{2}(2)(1) \\
 & - \frac{1}{2}(6)(4) \\
 & - \frac{1}{2}(2)(6) \\
 & = -17
 \end{aligned}$$

If the velocity of a particle at time t is given by $v(t) = t^2 - 9$, estimate the distance travelled by the object from $t = 3$ to $t = 9$ using 3 equal subintervals and the midpoint rule.

SCORE: ___ / 3 POINTS

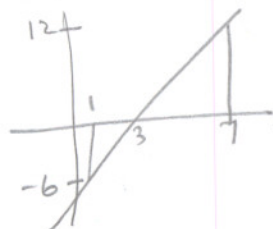
$$\begin{array}{c} 4 \quad 6 \quad 8 \\ | \quad | \quad | \quad | \\ 3 \quad 5 \quad 7 \quad 9 \end{array} \quad \Delta x = \frac{9-3}{3} = 2$$

$$(4^2 - 9)2 + (6^2 - 9)2 + (8^2 - 9)2 = 7 \cdot 2 + 27 \cdot 2 + 55 \cdot 2 = 178$$

OR

Evaluate $\int_1^7 (3x - 9) dx$ using geometry. Sketch a diagram and show the calculations used.

SCORE: ___ / 3 POINTS



$$\begin{aligned} & -\frac{1}{2}(6)(3-1) + \frac{1}{2}(12)(7-3) = -\frac{1}{2}(6)(2) + \frac{1}{2}(12)(4) \\ & = -6 + 24 \\ & = 18 \end{aligned}$$

OR

Write $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n} \arctan\left(4 + \frac{2k}{n}\right)$ as a definite integral. Do NOT evaluate the limit/definite integral.

SCORE: ___ / 3 POINTS

$$\int_{\frac{1}{4}}^{\frac{5}{4}} \arctan x \, dx$$

Write $\int_{-2}^{-4} g(x) dx - \int_4^2 g(x) dx + \int_{-4}^2 g(x) dx$ as a single integral in the form $\int_a^b g(x) dx$.

SCORE: ___ / 3 POINTS

Show each step CLEARLY.

SEE 7:30 VERSION D

If $\int_3^7 g(x) dx = -6$, find $\int_7^3 (2 + g(x)) dx$. Show each step CLEARLY.

SCORE: ___ / 4 POINTS

$$= -\int_3^7 (2 + g(x)) dx$$

$$= -\left[\int_3^7 2 dx + \int_3^7 g(x) dx\right]$$

$$= -[2(7-3) + -6]$$

$$= -[8 - 6] = -2$$

OR

$$\begin{aligned} & = \int_7^3 2 dx + \int_7^3 g(x) dx \\ & = 2(3-7) - \int_3^7 g(x) dx \\ & = -8 - (-6) \\ & = -2 \end{aligned}$$