Math 1A (7:30am – 8:20am)
Quiz 3 Version A
Fri Apr 22, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (USING THE THEOREMS IN 5.3 & 5.4)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus. **SCORE: ___ / 4 POINTS**
Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION L

The velocity of an object at time t (in seconds) is given by $v(t) = 1 - t^2$ meters per second. **SCORE: ___ / 5 POINTS**

[a] Find the displacement of the object from $t = 0$ to $t = 3$. Specify the units of your answer.

$$\underbrace{\int_0^3 (1-t^2)\,dt}_{1} = \underbrace{\left(t - \tfrac{1}{3}t^3\right)\Big|_0^3}_{1} = 3 - 9 = \underbrace{-6}_{\frac{1}{2}}\underbrace{m}_{\frac{1}{4}}$$

[b] Find the total distance travelled by the object from $t = 0$ to $t = 3$. Specify the units of your answer.

$$\int_0^3 |1-t^2|\,dt = \underbrace{\int_0^1 (1-t^2)\,dt + \int_1^3 -(1-t^2)\,dt}_{1\frac{1}{2}}$$

$$= \left(t - \tfrac{1}{3}t^3\right)\Big|_0^1 + -\left(t - \tfrac{1}{3}t^3\right)\Big|_1^3$$

$$= \left(1 - \tfrac{1}{3}\right) + -\left[(3-9) - \left(1 - \tfrac{1}{3}\right)\right] = \underbrace{\frac{22}{3}}_{\frac{1}{2}}\underbrace{m}_{\frac{1}{4}}$$

Find $\int_1^2 \frac{(2+r)^2}{4r^3}\,dr$. **SCORE: ___ / 5 POINTS**

$$= \int_1^2 \frac{4 + 4r + r^2}{4r^3}\,dr$$

$$= \underbrace{\int_1^2 \left(r^{-3} + r^{-2} + \tfrac{1}{4}r^{-1}\right)dr}_{2}$$

$$= \underbrace{\left(-\tfrac{1}{2}r^{-2} - r^{-1} + \tfrac{1}{4}\ln|r|\right)\Big|_1^2}$$ ← 2 POINTS IF ALL 3 TERMS CORRECT
1 ANY 2
0 0 OR 1

$$= \left(-\tfrac{1}{8} - \tfrac{1}{2} + \tfrac{1}{4}\ln 2\right) - \left(-\tfrac{1}{2} - 1\right)$$

$$= \underbrace{\frac{7}{8} + \frac{1}{4}\ln 2}_{1}$$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER **SCORE: ___ / 3 POINTS**

If you write $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(2+\frac{k}{n}\right)^{-3}$ as a definite integral, the value of the integral (and the limit) is $\int_2^3 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_2^3$

[a] $\frac{5}{72}$ [b] $\frac{1}{12}$ [c] $-\frac{1}{18}$ [d] $\frac{3}{8}$ [e] $\frac{7}{36}$ [f] none of the above

Find $\int (x^2+2x)\sin(x^3+3x^2-1)\,dx$. **SCORE: ___ / 4 POINTS**

$\underline{u = x^3+3x^2-1}$, $\frac{1}{2}$

$du = (3x^2+6x)\,dx$

$\frac{1}{3}\,du = (x^2+2x)\,dx$] $\frac{1}{2}$ ← ONLY NEED ONE OF THE TWO LINES TO GET THAT $\frac{1}{2}$ POINT

$\underbrace{\int \frac{1}{3}\sin u\,du}_{1} = \underbrace{-\frac{1}{3}\cos u}_{1} + \underbrace{C}_{\frac{1}{4}} = \underbrace{-\frac{1}{3}\cos(x^3+3x^2-1)}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{4}}$

Find the derivative of $\int_{x^4}^{\cosh x} \sqrt{t^2-1}\,dt$. **Show each step CLEARLY as demonstrated in class.** **SCORE: ___ / 4 POINTS**

$\frac{d}{dx}\int_{x^4}^{\cosh x} \sqrt{t^2-1}\,dt = \frac{d}{dx}\underbrace{\left[\int_{x^4}^{1}\sqrt{t^2-1}\,dt + \int_{1}^{\cosh x}\sqrt{t^2-1}\,dt\right]}_{}$, 1

$= \frac{d}{dx}\underbrace{\left[-\int_{1}^{x^4}\sqrt{t^2-1}\,dt + \int_{1}^{\cosh x}\sqrt{t^2-1}\,dt\right]}_{}$, $\frac{1}{2}$

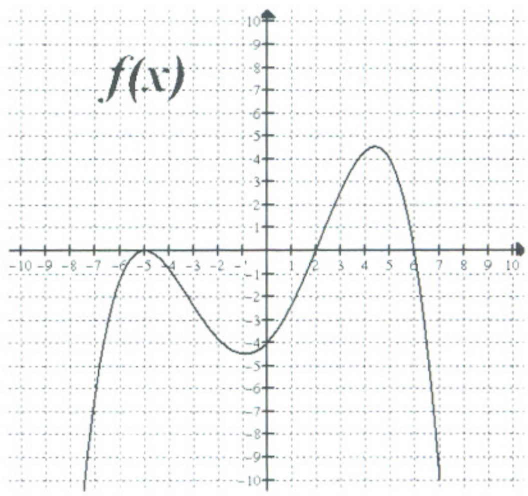
$= \underbrace{-\sqrt{x^8-1}}_{\frac{1}{2}} \cdot \underbrace{4x^3}_{\frac{1}{2}} + \underbrace{\sqrt{\cosh^2 x-1}}_{\frac{1}{2}} \cdot \underbrace{\sinh x}_{\frac{1}{2}}$

$= \underbrace{\sinh^2 x - 4x^3\sqrt{x^8-1}}_{\frac{1}{2}}$

The graph of f is shown on the right. Let $g(x) = \int_6^x f(t)\,dt$. **SCORE: ___ / 5 POINTS**

[a] Find $g'(5)$. **Justify your answer VERY BRIEFLY.**

1 $\underline{g'(5) = f(5)} = \underline{4}$, $\frac{1}{2}$



[b] At what value(s) of x does g have a local minimum (minima)?
Explain very briefly.

$\underline{g' (= f) \text{ CHANGES FROM } < 0 \text{ TO } > 0}$, 1

AT $\underline{x = 2}$, 1 ★ SUBTRACT $\frac{1}{2}$ POINT FOR EACH ADDITIONAL X-VALUE LISTED

[c] Is g concave up or concave down on the interval $(-7, -5)$?
Explain very briefly. Answers without explanations will earn no points.

$1\frac{1}{2}$ [$g' (= f)$ IS INCREASING ON $(-7, -5)$
SO g IS CONCAVE UP ← NO PARTIAL CREDIT IF EXPLANATION MISSING