Math 1A (7:30am – 8:20am)
Quiz 3 Version L
Fri Apr 22, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (USING THE THEOREMS IN 5.3 & 5.4)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus. **SCORE: ___ / 4 POINTS**
Use complete sentences and proper algebra & English as shown in class.

IF f IS CONTINUOUS AND $g(x)=\int_a^x f(t)\,dt$, THEN $g'(x)=f(x)$

IF f IS CONTINUOUS ON $[a,b]$ AND F IS ANY ANTIDERIVATIVE OF f, THEN $\int_a^b f(x)\,dx = F(b)-F(a)$

The velocity of an object at time t (in seconds) is given by $v(t)=12-3t^2$ meters per second. **SCORE: ___ / 5 POINTS**

[a] Find the displacement of the object from $t=0$ to $t=5$. Specify the units of your answer.

$\underbrace{\int_0^5 (12-3t^2)\,dt}_{1} = \underbrace{(12t-t^3)\Big|_0^5}_{1} = 60-125 = \underbrace{-65}_{\frac{1}{2}}\ \underbrace{m}_{\frac{1}{4}}$

[b] Find the total distance travelled by the object from $t=0$ to $t=5$. Specify the units of your answer.

$\int_0^5 |12-3t^2|\,dt = \underbrace{\int_0^2 (12-3t^2)\,dt + \int_2^5 -(12-3t^2)\,dt}_{1\frac{1}{2}}$

$= (12t-t^3)\Big|_0^2 + -(12t-t^3)\Big|_2^5$

$= (24-8) + -[(60-125)-(24-8)] = \underbrace{97}_{\frac{1}{2}}\ \underbrace{m}_{\frac{1}{4}}$

Find $\int_1^2 \frac{(1+r)^2}{2r^3}\,dr$. **SCORE: ___ / 5 POINTS**

$= \int_1^2 \frac{1+2r+r^2}{2r^3}\,dr$

$= \underbrace{\int_1^2 \left(\frac{1}{2}r^{-3}+r^{-2}+\frac{1}{2}r^{-1}\right)dr}_{2}$

$= \underbrace{\left(-\frac{1}{4}r^{-2}-r^{-1}+\frac{1}{2}\ln|r|\right)\Big|_1^2}_{2}$ ←

$= \left(-\frac{1}{16}-\frac{1}{2}+\frac{1}{2}\ln 2\right)-\left(-\frac{1}{4}-1\right)$

$= \underbrace{\frac{11}{16}+\frac{1}{2}\ln 2}_{1}$

0 POINTS IF 0 OR 1 TERMS CORRECT
1 ANY 2
2 ALL 3

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER **SCORE: ___ / 3 POINTS**

If you write $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{3}{n}\left(1+\frac{3k}{n}\right)^{-3}$ as a definite integral, the value of the integral (and the limit) is

$\int_1^4 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^4$

[a] $\frac{7}{16}$ [b] $-\frac{31}{64}$ [c] $\frac{3}{8}$ [d] $-\frac{4}{9}$ [e] $\frac{15}{32}$ [f] none of the above

Find $\int (x^2+2x)\sec^2(x^3+3x^2-1)\,dx$. **SCORE: ___ / 4 POINTS**

$u = x^3+3x^2-1$ ½

$du = (3x^2+6x)\,dx$

$\frac{1}{3}du = (x^2+2x)\,dx$] ½ ← ONLY NEED ONE OF THE TWO LINES TO GET THE ½ POINT

$\int \frac{1}{3}\sec^2 u\,du = \frac{1}{3}\tan u + C = \frac{1}{3}\tan(x^3+3x^2-1) + C$

1 1 ¼ ½ ¼

Find the derivative of $\int_{\cosh x}^{x^2}\sqrt{t^2-1}\,dt$. **Show each step CLEARLY as demonstrated in class.** **SCORE: ___ / 4 POINTS**

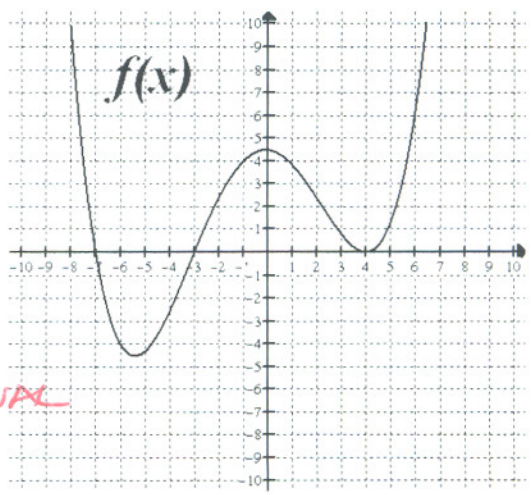
$\frac{d}{dx}\int_{\cosh x}^{x^2}\sqrt{t^2-1}\,dt = \frac{d}{dx}\left[\int_{\cosh x}^{1}\sqrt{t^2-1}\,dt + \int_1^{x^2}\sqrt{t^2-1}\,dt\right]$ 1

$= \frac{d}{dx}\left[-\int_1^{\cosh x}\sqrt{t^2-1}\,dt + \int_1^{x^2}\sqrt{t^2-1}\,dt\right]$ ½

$= \underbrace{-\sqrt{\cosh^2 x-1}}_{\frac{1}{2}}\cdot\underbrace{\sinh x}_{\frac{1}{2}} + \underbrace{\sqrt{x^4-1}}_{\frac{1}{2}}\cdot\underbrace{2x}_{\frac{1}{2}}$

$= 2x\sqrt{x^4-1} - \sinh^2 x$ ½

The graph of f is shown on the right. Let $g(x) = \int_{-1}^{x} f(t)\,dt$. **SCORE: ___ / 5 POINTS**



[a] Find $g'(-6)$. **Justify your answer VERY BRIEFLY.**

1 $g'(-6) = f(-6) = -4$ ½

[b] At what value(s) of x does g have a local maximum (maxima)?
Explain very briefly.

$g'(=f)$ CHANGES FROM >0 TO <0 1

AT $x=-7$ 1

★ SUBTRACT ½ POINT FOR EACH ADDITIONAL X-VALUE LISTED

[c] Is g concave up or concave down on the interval $(2,4)$?
Explain very briefly. Answers without explanations will earn no points.

1½ [$g'(=f)$ IS DECREASING ON $(2,4)$

SO g IS CONCAVE DOWN ← NO PARTIAL CREDIT IF EXPLANATION MISSING