Math 1A (9:30am – 10:20am)
Quiz 3 Version D
Fri Apr 22, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (USING THE THEOREMS IN 5.3 & 5.4)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus. **SCORE: ___ / 4 POINTS**
Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION L

The velocity of an object at time t (in seconds) is given by $v(t) = 4 - t^2$ meters per second. **SCORE: ___ / 5 POINTS**

[a] Find the displacement of the object from $t = 0$ to $t = 3$. Specify the units of your answer.

$$\underbrace{\int_0^3 (4-t^2)\,dt}_{1} = \underbrace{\left(4t-\tfrac{1}{3}t^3\right)\Big|_0^3}_{1} = 12-9 = \underbrace{3}_{\frac{1}{2}}\,\underbrace{m}_{\frac{1}{4}}$$

[b] Find the total distance travelled by the object from $t = 0$ to $t = 3$. Specify the units of your answer.

$$\int_0^3 |4-t^2|\,dt = \underbrace{\int_0^2 (4-t^2)\,dt + \int_2^3 -(4-t^2)\,dt}_{}, \ 1\tfrac{1}{2}$$

$$= \left(4t-\tfrac{1}{3}t^3\right)\Big|_0^2 + -\left(4t-\tfrac{1}{3}t^3\right)\Big|_2^3$$

$$= \left(8-\tfrac{8}{3}\right) + -\left[(12-9)-\left(8-\tfrac{8}{3}\right)\right] = \underbrace{\tfrac{23}{3}}_{\frac{1}{2}}\,\underbrace{m}_{\frac{1}{4}}$$

Find $\int_1^2 \frac{(3+r)^2}{3r^3}\,dr$. **SCORE: ___ / 5 POINTS**

$$= \int_1^2 \frac{9+6r+r^2}{3r^3}\,dr$$

$$= \underbrace{\int_1^2 \left(3r^{-3}+2r^{-2}+\tfrac{1}{3}r^{-1}\right)dr}_{}, \ 2$$

$$= \underbrace{\left(-\tfrac{3}{2}r^{-2}-2r^{-1}+\tfrac{1}{3}\ln|r|\right)\Big|_1^2}_{} \leftarrow$$

0 POINTS IF 0 OR 1 TERM CORRECT
1 ANY 2
2 ALL 3

$$= \left(-\tfrac{3}{8}-1+\tfrac{1}{3}\ln 2\right) - \left(-\tfrac{3}{2}-2\right)$$

$$= \underbrace{\tfrac{17}{8}+\tfrac{1}{3}\ln 2}_{}, \ 1$$

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER **SCORE: ___ / 3 POINTS**

If you write $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n}\left(1+\frac{2k}{n}\right)^{-4}$ as a definite integral, the value of the integral (and the limit) is $\int_1^3 x^{-4}\,dx = -\frac{1}{3}x^{-3}\Big|_1^3$

[a] $\frac{10}{27}$ [b] $\frac{26}{81}$ [c] $-\frac{1}{3}$ [d] $\frac{8}{27}$ [e] $\frac{2}{9}$ [f] none of the above

Find $\int (x^3+x)\csc^2(x^4+2x^2-5)\,dx$. **SCORE: ___ / 4 POINTS**

$\underbrace{u = x^4+2x^2-5}_{\frac{1}{2}}$

$du = (4x^3+4x)\,dx$

$\frac{1}{4}du = (x^3+x)\,dx$ $\Big]\ \frac{1}{2}$ ← ONLY NEED ONE OF THESE TWO LINES TO GET THE $\frac{1}{2}$ POINT

$\underbrace{\int \frac{1}{4}\csc^2 u\,du}_{1} = \underbrace{-\frac{1}{4}\cot u}_{1} + \underbrace{C}_{\frac{1}{4}} = \underbrace{-\frac{1}{4}\cot(x^4+2x^2-5)}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{4}}$

Find the derivative of $\int_{x^2}^{\sinh x}\sqrt{t^2+1}\,dt$. **Show each step CLEARLY as demonstrated in class.** **SCORE: ___ / 4 POINTS**

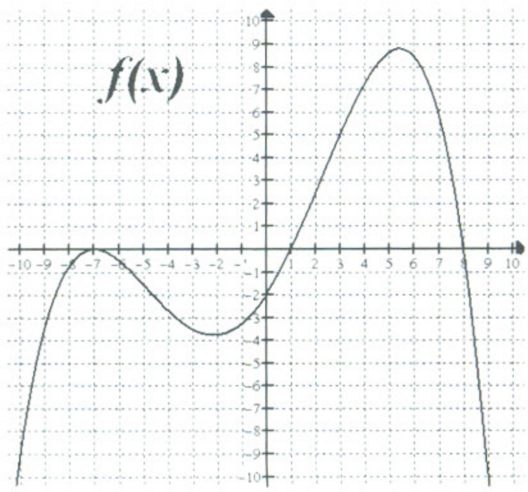
$$\frac{d}{dx}\int_{x^2}^{\sinh x}\sqrt{t^2+1}\,dt = \frac{d}{dx}\Big[\underbrace{\int_{x^2}^{0}\sqrt{t^2+1}\,dt + \int_0^{\sinh x}\sqrt{t^2+1}\,dt}_{1}\Big]$$

$$= \frac{d}{dx}\Big[\underbrace{-\int_0^{x^2}\sqrt{t^2+1}\,dt + \int_0^{\sinh x}\sqrt{t^2+1}\,dt}_{\frac{1}{2}}\Big]$$

$$= \underbrace{-\sqrt{x^4+1}}_{\frac{1}{2}}\cdot\underbrace{2x}_{\frac{1}{2}} + \underbrace{\sqrt{\sinh^2 x+1}}_{\frac{1}{2}}\cdot\underbrace{\cosh x}_{\frac{1}{2}}$$

$$= \underbrace{\cosh^2 x - 2x\sqrt{x^4+1}}_{\frac{1}{2}}$$

The graph of f is shown on the right. Let $g(x) = \int_8^x f(t)\,dt$. **SCORE: ___ / 5 POINTS**



[a] Find $g'(3)$. **Justify your answer VERY BRIEFLY.**

$1\ \underbrace{g'(3) = f(3)}_{} = \underbrace{5}_{\frac{1}{2}}$

[b] At what value(s) of x does g have a local minimum (minima)?
Explain very briefly.

1 $g'(=f)$ CHANGES FROM <0 TO >0

AT $\underbrace{x=1}_{1}$ ★ SUBTRACT $\frac{1}{2}$ POINT FOR EACH ADDITIONAL X-VALUE LISTED

[c] Is g concave up or concave down on the interval $(-9, -7)$?
Explain very briefly. Answers without explanations will earn no points.

$1\frac{1}{2}$ [$g'(=f)$ IS INCREASING ON $(-9,-7)$

SO g IS CONCAVE UP ← NO PARTIAL CREDIT IF EXPLANATION MISSING