

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK (USING THE THEOREMS IN 5.3 & 5.4)
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

State both parts of the Fundamental Theorem of Calculus.

SCORE: ___ / 4 POINTS

Use complete sentences and proper algebra & English as shown in class.

SEE 7:30 VERSION L

The velocity of an object at time t (in seconds) is given by $v(t) = 3 - 3t^2$ meters per second.

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[a] Find the displacement of the object from $t = 0$ to $t = 5$. Specify the units of your answer.

$$\int_0^5 (3 - 3t^2) dt = (3t - t^3) \Big|_0^5 = 15 - 125 = -110 \text{ m}$$

$\underbrace{\hspace{100px}}_{\frac{1}{2}}$
 $\underbrace{\hspace{100px}}_{\frac{1}{4}}$

[b] Find the total distance travelled by the object from $t = 0$ to $t = 5$. Specify the units of your answer.

$$\begin{aligned} \int_0^5 |3 - 3t^2| dt &= \int_0^1 (3 - 3t^2) dt + \int_1^5 -(3 - 3t^2) dt, \frac{1}{2} \\ &= (3t - t^3) \Big|_0^1 + -(3t - t^3) \Big|_1^5 \\ &= (3 - 1) + -[(15 - 125) - (3 - 1)] = 114 \text{ m} \end{aligned}$$

$\underbrace{\hspace{100px}}_{\frac{1}{2}}$
 $\underbrace{\hspace{100px}}_{\frac{1}{4}}$

Find $\int_1^2 \frac{(1-r)^2}{2r^3} dr$.

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$$\begin{aligned} &= \int_1^2 \frac{1 - 2r + r^2}{2r^3} dr \\ &= \int_1^2 \left(\frac{1}{2} r^{-3} - r^{-2} + \frac{1}{2} r^{-1} \right) dr \\ &= \left(-\frac{1}{4} r^{-2} + r^{-1} + \frac{1}{2} \ln |r| \right) \Big|_1^2 \leftarrow \\ &= \left(-\frac{1}{16} + \frac{1}{2} + \frac{1}{2} \ln 2 \right) - \left(-\frac{1}{4} + 1 \right) \\ &= -\frac{5}{16} + \frac{1}{2} \ln 2 \end{aligned}$$

2 POINTS IF ALL 3 TERMS CORRECT
 1 ANY 2
 0 or 1

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER

SCORE: ___ / 3 POINTS

If you write $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3}{n} \left(1 + \frac{3k}{n}\right)^{-4}$ as a definite integral, the value of the integral (and the limit) is

$$\int_1^4 x^{-4} dx = -\frac{1}{3} x^{-3} \Big|_1^4$$

- [a] $\frac{3}{8}$ [b] $\frac{65}{192}$ [c] $-\frac{1}{3}$ [d] $\frac{5}{16}$ [e] $\frac{21}{64}$ [f] none of the above

Find $\int (x^3 + x) \sin(x^4 + 2x^2 - 5) dx$.

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$u = x^4 + 2x^2 - 5$ $\frac{1}{2}$
 $du = (4x^3 + 4x) dx$
 $\frac{1}{4} du = (x^3 + x) dx$

NEED ONLY ONE OF THESE TWO LINES TO GET THAT $\frac{1}{2}$ POINT

$\int \frac{1}{4} \sin u du = -\frac{1}{4} \cos u + C = -\frac{1}{4} \cos(x^4 + 2x^2 - 5) + C$

Find the derivative of $\int_{\sinh x}^{x^4} \sqrt{t^2 + 1} dt$. Show each step CLEARLY as demonstrated in class.

SCORE: ___ / 4 POINTS

$\frac{d}{dx} \int_{\sinh x}^{x^4} \sqrt{t^2 + 1} dt = \frac{d}{dx} \left[\int_{\sinh x}^0 \sqrt{t^2 + 1} dt + \int_0^{x^4} \sqrt{t^2 + 1} dt \right]$
 $= \frac{d}{dx} \left[- \int_0^{\sinh x} \sqrt{t^2 + 1} dt + \int_0^{x^4} \sqrt{t^2 + 1} dt \right]$
 $= -\sqrt{\sinh^2 x + 1} \cdot \cosh x + \sqrt{x^8 + 1} \cdot 4x^3$
 $= 4x^3 \sqrt{x^8 + 1} - \cosh^2 x$

The graph of f is shown on the right. Let $g(x) = \int_{-1}^x f(t) dt$.

SCORE: ___ / 5 POINTS

[a] Find $g'(-2)$. Justify your answer VERY BRIEFLY.

$g'(-2) = f(-2) = -5$ $\frac{1}{2}$

[b] At what value(s) of x does g have a local maximum (maxima)?
Explain very briefly.

$g' (= f)$ CHANGES FROM > 0 TO < 0
 AT $x = -7$ $\star$ SUBTRACT $\frac{1}{2}$ POINT FOR EACH ADDITIONAL x -VALUE LISTED

[c] Is g concave up or concave down on the interval $(6, 8)$?

Explain very briefly. Answers without explanations will earn no points.

$g' (= f)$ IS DECREASING ON $(6, 8)$
 SO g IS CONCAVE DOWN

NO PARTIAL CREDIT IF EXPLANATION MISSING

