Math 1B (7:30am – 8:20am)
Quiz 5 Version 8
Fri May 13, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER **SCORE: ___ / 3 POINTS**

A 5 foot long chain weighing 20 pounds hangs from a hook in the ceiling of an 11 foot tall room. (So, the bottom of the chain is 6 feet from the floor.) How many foot-pounds of work are done lifting the bottom loop of the chain to the ceiling so that it touches the top loop ?
(HINT: Draw "before" and "after" diagrams.)

[a] 25 [b] 10 [c] 20 [d] 30 [e] 15

A 50 foot chain weighing 4 pounds per foot hangs over the edge of a 50 foot tall building. The chain is used to lift a 25 pound tabletop from ground level to a window 20 feet above ground. **SCORE: ___ / 6 POINTS**
Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for the work done.

IF $x=0$ IS ROOF AND $x=50$ IS GROUND

$$(25)(20) + \int_{30}^{50} 4x\,dx$$

OR

$$\int_{0}^{20} (25 + 4(50-x))\,dx$$

OR IF $x=50$ IS ROOF AND $x=0$ IS GROUND

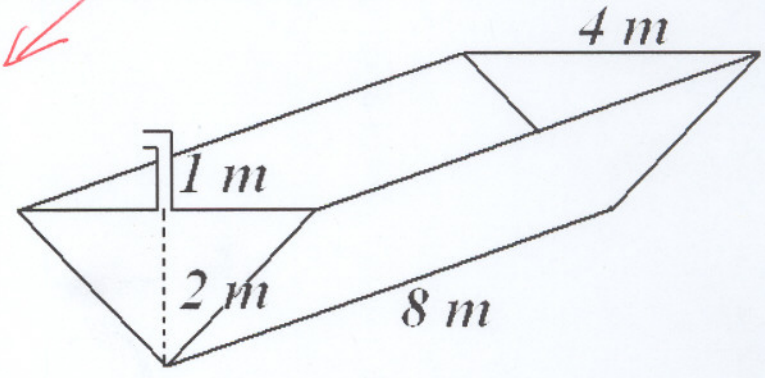
$$(25)(20) + \int_{0}^{20} 4(50-x)\,dx$$

OR

$$\int_{30}^{50} (25 + 4x)\,dx$$

TALK TO ME IF YOU USED ANY OTHER SCALE FOR X

A tank in the shape of the triangular prism shown on the right is filled with water. **SCORE: ___ / 6 POINTS**
Write, **BUT DO NOT EVALUATE**, an integral for the work required to pump the water out of the spout.



IF $x=0$ IS BOTTOM OF TANK AND $x=2$ IS TOP OF TANK AND $x=3$ IS TOP OF SPOUT

$$\int_{0}^{2} (9.8)(1000)(8(2x))(3-x)\,dx$$

OR

IF $x=0$ IS TOP OF TANK AND $x=2$ IS BOTTOM OF TANK AND $x=-1$ IS TOP OF SPOUT

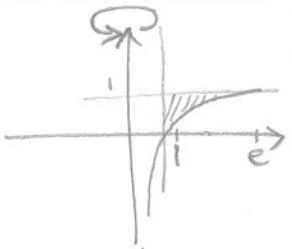
$$\int_{0}^{2} (9.8)(1000)(8(4-2x))(x+1)\,dx$$

OR IF $x=0$ IS TOP OF SPOUT AND $x=1$ IS TOP OF TANK AND $x=3$ IS BOTTOM OF TANK

$$\int_{1}^{3} (9.8)(1000)(8(6-2x))(x)\,dx$$

The region bounded by $x=1$, $y=\ln x$ and $y=1$ is revolved around the $y-$axis. **SCORE: ___ / 6 POINTS**

Find the volume of the solid.



$y=\ln x$

$x=e^y$

$\frac{1}{2}$ $\int_0^1 \pi\left((e^y)^2 - 1^2\right)dy$ OR $\int_{\frac{1}{2}}^{1} 2\pi x(1-\ln x)dx + \int_1^e 2\pi x(1-\ln x)dx$

$\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $1\frac{1}{2}$

$= \pi\int_0^1 (e^{2y}-1)dy$

$= \pi\left(\frac{1}{2}e^{2y}-y\right)\Big|_0^1$

$= \pi\left(\frac{1}{2}e^2-1-\frac{1}{2}\right)$

$= \pi\left(\frac{1}{2}e^2-\frac{3}{2}\right)$

$= \frac{\pi}{2}(e^2-3)$ $\frac{1}{2}$

$= 2\pi\int_1^e (x-x\ln x)dx$

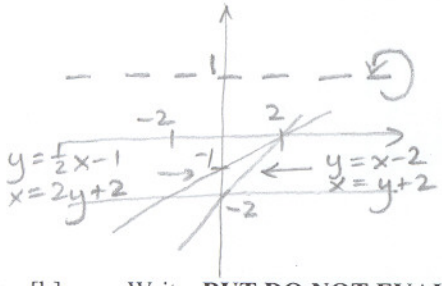
$= 2\pi\left(\frac{1}{4}x^2(3-2\ln x)\right)\Big|_1^e$

$= 2\pi\left(\frac{1}{4}e^2-\frac{3}{4}\right)$

$= \frac{\pi}{2}(e^2-3)$ $\frac{1}{2}$

The region bounded by $y=-2$, $y=\frac{1}{2}x-1$ and $y=x-2$ is revolved around the line $y=1$. **SCORE: ___ / 9 POINTS**

[a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.



$y=\frac{1}{2}x-1$ x-INT $(2,0)$ y-INT $(0,-1)$ $\frac{1}{2}x-1=-2 \to x=-2$

$y=x-2$ x-INT $(2,0)$ y-INT $(0,-2)$ $x-2=-2 \to x=0$

$\frac{1}{2}$ $\int_{-2}^{0}$ $\frac{1}{2}$ $2\pi(1-y)(y+2-(2y+2))dy$

$= \int_{-2}^{0} 2\pi(1-y)(-y)dy = \int_{-2}^{0} 2\pi(y^2-y)dy$

[b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the washer method.

$\int_{-2}^{0} \pi\left(3^2-\left(1-\left(\frac{1}{2}x-1\right)^2\right)\right)dx + \int_0^2 \pi\left(\left(1-(x-2)^2\right)^2-\left(1-\left(\frac{1}{2}x-1\right)^2\right)^2\right)dx$

$\frac{1}{4}$ $\int_{-2}^{0}$ π $\left(9 - \left(2-\frac{1}{2}x\right)^2\right)dx + \frac{1}{4}\int_0^2 \pi\left((3-x)^2 - \left(2-\frac{1}{2}x\right)^2\right)dx$

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{4}$

[c] Find the volume of the solid by evaluating the appropriate integral(s) from either [a] or [b].

$\int_{-2}^{0} 2\pi(y^2-y)dy$

$= 2\pi\left(\frac{1}{3}y^3-\frac{1}{2}y^2\right)\Big|_{-2}^{0}$

$= 2\pi\left(0-\left(-\frac{8}{3}-2\right)\right)$

$= \frac{28\pi}{3}$ $\frac{1}{2}$