

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED

**SHOW PROPER ALGEBRAIC WORK
 USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE**

MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER

SCORE: ____ / 3 POINTS

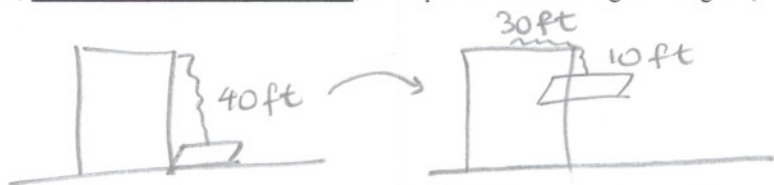
A 5 foot long chain weighing 24 pounds hangs from a hook in the ceiling of an 11 foot tall room. (So, the bottom of the chain is 6 feet from the floor.) How many foot-pounds of work are done lifting the bottom loop of the chain to the ceiling so that it touches the top loop?
 (HINT: Draw "before" and "after" diagrams.)

- [a] 25 [b] 10 [c] 20 [d] 30 [e] 15

A 40 foot chain weighing 3 pounds per foot hangs over the edge of a 40 foot tall building. The chain is used to lift a 15 pound tabletop from ground level to a window 30 feet above ground.

SCORE: ____ / 6 POINTS

Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for the work done.



IF $x=0$ IS ROOF OR IF $x=40$ IS ROOF
 AND $x=40$ IS GROUND AND $x=0$ IS GROUND

$$\frac{(15)(30)}{2} + \int_{10}^{40} 3x \, dx$$

OR

$$\int_0^{30} (15 + 3(40-x)) \, dx$$

OR

$$\frac{(15)(30)}{2} + \int_0^{30} 3(40-x) \, dx$$

OR

$$\int_{10}^{40} (15 + 3x) \, dx$$

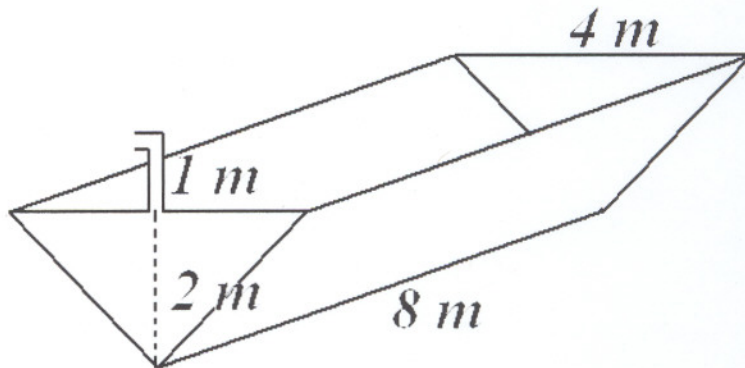
TALK TO ME
 IF YOU USED
 ANY OTHER
 SCALE FOR x

A tank in the shape of the triangular prism shown on the right is filled with water.

SCORE: ____ / 6 POINTS

Write, **BUT DO NOT EVALUATE**, an integral for the work required to pump the water out of the spout.

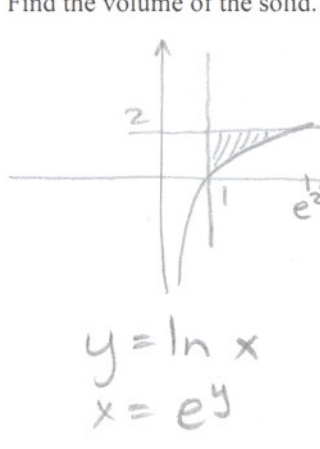
SEE 7:30 VERSION 8



The region bounded by $x = 1$, $y = \ln x$ and $y = 2$ is revolved around the y -axis.

SCORE: ___ / 6 POINTS

Find the volume of the solid.



$y = \ln x$
 $x = e^y$

$$\begin{aligned} \frac{1}{2} \int_0^2 \pi ((e^y)^2 - 1^2) dy &= \int_0^2 \pi (e^{2y} - 1) dy \\ &= \pi \left(\frac{1}{2} e^{2y} - y \right) \Big|_0^2 \\ &= \pi \left(\frac{1}{2} e^4 - 2 - \frac{1}{2} \right) \\ &= \pi \left(\frac{1}{2} e^4 - \frac{5}{2} \right) \\ &= \frac{\pi}{2} (e^4 - 5) \end{aligned}$$

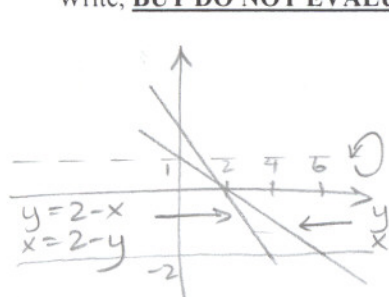
OR

$$\begin{aligned} \int_{\frac{1}{2}}^1 \int_{\frac{1}{2}}^{e^2} 2\pi x (2 - \ln x) dx &= 2\pi \int_1^{e^2} (2x - x \ln x) dx \\ &= 2\pi \left(\frac{1}{2} x^2 (5 - 2 \ln x) \right) \Big|_1^{e^2} \\ &= 2\pi \left(\frac{1}{4} e^4 - \frac{5}{4} \right) \\ &= \frac{\pi}{2} (e^4 - 5) \end{aligned}$$

The region bounded by $y = -2$, $y = 1 - \frac{1}{2}x$ and $y = 2 - x$ is revolved around the line $y = 1$.

SCORE: ___ / 9 POINTS

[a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.



$y = 1 - \frac{1}{2}x$ $x = 2 - 2y$ $y = 2 - x$ $x = 2 - 2y$

$$\int_{-2}^0 2\pi (1-y) (2-2y - (2-y)) dy = \int_{-2}^0 2\pi (1-y)(-y) dy = \int_{-2}^0 2\pi (y^2 - y) dy$$

[b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the washer method.

$$\begin{aligned} \int_2^4 \pi ((1 - (2-x))^2 - (1 - (1 - \frac{1}{2}x))^2) dx + \int_4^6 \pi ((3 - (1 - (1 - \frac{1}{2}x)))^2) dx \\ = \int_{\frac{1}{4}}^{\frac{1}{2}} \pi ((x-1)^2 - \frac{1}{4}x^2) dx + \int_{\frac{1}{4}}^{\frac{1}{2}} \pi (9 - \frac{1}{4}x^2) dx \end{aligned}$$

[c] Find the volume of the solid by evaluating the appropriate integral(s) from either [a] or [b].

$$\begin{aligned} \int_{-2}^0 2\pi (y^2 - y) dy &= 2\pi \left(\frac{1}{3} y^3 - \frac{1}{2} y^2 \right) \Big|_{-2}^0 \\ &= 2\pi \left(0 - \left(-\frac{8}{3} - 2 \right) \right) \\ &= \frac{28\pi}{3} \end{aligned}$$