Explanation of some solutions from Quiz 5

The multiple choice question

BEFORE	**AFTER**
O	OO
O	OO
O	OO
O	OO
O	
O	
O	
O	

NET DIFFERENCE

bottom half of the chain all lifted exactly half the length of the chain

WORK DONE

(½ × total weight of the chain) × (½ × total length of the chain).

The chain & tabletop question

Solution for the following version of the problem:

> A 50 foot chain weighing 4 pounds per foot hangs over the edge of a 50 foot tall building.
> The chain is used to lift a 25 pound tabletop from ground level to a window 20 feet above ground.

BEFORE	**AFTER**
50 feet of chain hanging from roof tabletop on ground	20 feet of chain on roof 30 feet of chain hanging from roof tabletop 20 feet above ground

BEFORE	AFTER
_____	___OO
O	O
O	O
O	O
O	TABLETOP
O	
_____ TABLETOP ON GROUND	_____ GROUND

NET DIFFERENCE

bottom 20 feet of the chain lifted various distances to the roof
tabletop lifted 20 feet

WORK DONE ON TABLETOP

$W_{tabletop}$ = weight of the tabletop × distance moved by the tabletop
= 25 pounds × 20 feet

WORK DONE ON CHAIN

each small piece of chain had length Δx feet and density 4 pounds per foot
so $F_{1\ piece} = 4\Delta x$ pounds

looking at the 20 feet of chain that was moved to the roof	**if x = 0 at the roof and x = 50 at the ground**	**if x = 0 at the ground and x = 50 at the roof**
top end was originally at **<u>and was moved d = 30 feet to the roof</u>**	x = 30	x = 20
bottom end was originally at **<u>and was moved d = 50 feet to the roof</u>**	x = 50	x = 0
	$d_{1\ piece}$ = x feet $W_{1\ piece} = F_{1\ piece} d_{1\ piece}$ $= 4\Delta x$ pounds × x feet $W_{chain} = \int_{30}^{50} 4x\,dx$	$d_{1\ piece} = (50 - x)$ ft $W_{1\ piece} = F_{1\ piece} d_{1\ piece}$ $= 4\Delta x$ pounds × (50 – x) feet $W_{chain} = \int_{0}^{20} 4(50 - x)\,dx$
TOTAL WORK DONE = $W_{tabletop} + W_{chain}$	$= 25 \times 20 + \int_{30}^{50} 4x\,dx$	$= 25 \times 20 + \int_{0}^{20} 4(50 - x)\,dx$

The water tank question

Solution for the version of the question with the 1 m tall spout:

"slicing" the water horizontally creates slices which look like rectangular prisms
up-and-down height of each slide is Δx m
front-to-back length of each slice is 8 m

if top of spout at **and top of tank at** **and bottom of tank at**	**$x = 3$** **$x = 2$** **$x = 0$**	**$x = -1$** **$x = 0$** **$x = 2$**	**$x = 0$** **$x = 1$** **$x = 3$**
top slice was originally at **and had a side-to-side width of 4 m** **and was moved 1 m to the top of the spout**	$x = 2$	$x = 0$	$x = 1$
bottom slice was originally at **and had a side-to-side width of 0 m** **and was moved 3 m to the top of the spout**	$x = 0$	$x = 2$	$x = 3$
side-to-side width of 1 slice = distance moved by 1 slice =	$2x$ m $d_{1\ slice} = (3 - x)$ m	$(4 - 2x)$ m $d_{1\ slice} = (x + 1)$ m	$(6 - 2x)$ m $d_{1\ slice} = x$ m
$volume_{1\ slice}$ =	$(8)(2x)(\Delta x)\ m^3$	$(8)(4 - 2x)(\Delta x)\ m^3$	$(8)(6 - 2x)(\Delta x)\ m^3$

$mass_{1\ slice} = volume_{1\ slice} \times$ density of water ($1000\ kg/m^3$)

$F_{1\ slice} = mass_{1\ slice} \times$ acceleration due to gravity ($9.8\ m/s^2$)

$W_{1\ slice} = F_{1\ slice} d_{1\ slice} =$

$(9.8)(1000)(8)(2x)(3 - x)(\Delta x)$
$(9.8)(1000)(8)(4 - 2x)(x + 1)(\Delta x)$
$(9.8)(1000)(8)(6 - 2x)(x)(\Delta x)$

WORK DONE

$$= \int_0^2 9.8(1000)(8)(2x)(3-x)\,dx$$

$$= \int_0^2 9.8(1000)(8)(4-2x)(x+1)\,dx$$

$$= \int_1^3 9.8(1000)(8)(6-2x)(x)\,dx$$