

Explanation of some solutions from Quiz 5

The multiple choice question

BEFORE AFTER

<hr/> O	<hr/> OO
O	OO
O	OO
O	OO
O	
O	
O	
O	
O	

NET DIFFERENCE

bottom half of the chain all lifted exactly half the length of the chain

WORK DONE

$(\frac{1}{2} \times \text{total weight of the chain}) \times (\frac{1}{2} \times \text{total length of the chain}).$

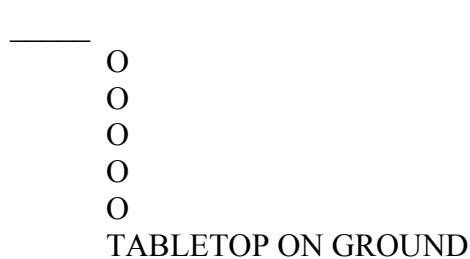
The chain & tabletop question

Solution for the following version of the problem:

A 50 foot chain weighing 4 pounds per foot hangs over the edge of a 50 foot tall building.
The chain is used to lift a 25 pound tabletop from ground level to a window 20 feet above ground.

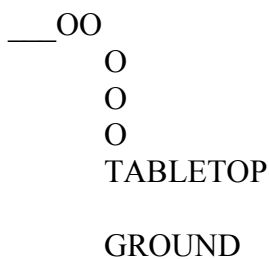
BEFORE

50 feet of chain hanging from roof
tabletop on ground



AFTER

20 feet of chain on roof
30 feet of chain hanging from roof
tabletop 20 feet above ground



NET DIFFERENCE

bottom 20 feet of the chain lifted various distances to the roof
tabletop lifted 20 feet

WORK DONE ON TABLETOP

$W_{\text{tabletop}} = \text{weight of the tabletop} \times \text{distance moved by the tabletop}$
 $= 25 \text{ pounds} \times 20 \text{ feet}$

WORK DONE ON CHAIN

each small piece of chain had length Δx feet and density 4 pounds per foot
so $F_{\text{1 piece}} = 4\Delta x$ pounds

looking at the 20 feet of chain
that was moved to the roof

**if $x = 0$ at the roof
and $x = 50$ at the ground**

**if $x = 0$ at the ground
and $x = 50$ at the roof**

top end was originally at $x = 30$
and was moved $d = 30$ feet to the roof

$x = 20$

bottom end was originally at $x = 50$
and was moved $d = 50$ feet to the roof

$x = 0$

$$\begin{aligned} d_{\text{1 piece}} &= x \text{ feet} \\ W_{\text{1 piece}} &= F_{\text{1 piece}} d_{\text{1 piece}} \\ &= 4\Delta x \text{ pounds} \times x \text{ feet} \end{aligned}$$

$$W_{\text{chain}} = \int_{30}^{50} 4x \, dx$$

$$\text{TOTAL WORK DONE} = W_{\text{tabletop}} + W_{\text{chain}} = 25 \times 20 + \int_{30}^{50} 4x \, dx$$

$$\begin{aligned} d_{\text{1 piece}} &= (50 - x) \text{ ft} \\ W_{\text{1 piece}} &= F_{\text{1 piece}} d_{\text{1 piece}} \\ &= 4\Delta x \text{ pounds} \times (50 - x) \text{ feet} \end{aligned}$$

$$W_{\text{chain}} = \int_0^{20} 4(50 - x) \, dx$$

$$= 25 \times 20 + \int_0^{20} 4(50 - x) \, dx$$

The water tank question

Solution for the version of the question with the 1 m tall spout:

“slicing” the water horizontally creates slices which look like rectangular prisms

up-and-down height of each slice is Δx m

front-to-back length of each slice is 8 m

if top of spout at
and top of tank at
and bottom of tank at

$x = 3$
 $x = 2$
 $x = 0$

$x = -1$
 $x = 0$
 $x = 2$

$x = 0$
 $x = 1$
 $x = 3$

top slice was originally at
and had a side-to-side width of 4 m
and was moved 1 m to the top of the spout

$x = 2$

$x = 0$

$x = 1$

bottom slice was originally at
and had a side-to-side width of 0 m
and was moved 3 m to the top of the spout

$x = 0$

$x = 2$

$x = 3$

side-to-side width of 1 slice =
distance moved by 1 slice =

$2x$ m
 $d_{1 \text{ slice}} = (3 - x)$ m

$(4 - 2x)$ m
 $d_{1 \text{ slice}} = (x + 1)$ m

$(6 - 2x)$ m
 $d_{1 \text{ slice}} = x$ m

$\text{volume}_{1 \text{ slice}} =$

$(8)(2x)(\Delta x) \text{ m}^3$

$(8)(4 - 2x)(\Delta x) \text{ m}^3$

$(8)(6 - 2x)(\Delta x) \text{ m}^3$

$\text{mass}_{1 \text{ slice}} = \text{volume}_{1 \text{ slice}} \times \text{density of water (1000 kg/m}^3\text{)}$

$F_{1 \text{ slice}} = \text{mass}_{1 \text{ slice}} \times \text{acceleration due to gravity (9.8 m/s}^2\text{)}$

$W_{1 \text{ slice}} = F_{1 \text{ slice}} d_{1 \text{ slice}} =$

$(9.8)(1000)(8)(2x)(3 - x)(\Delta x)$
 $(9.8)(1000)(8)(4 - 2x)(x + 1)(\Delta x)$
 $(9.8)(1000)(8)(6 - 2x)(x)(\Delta x)$

WORK DONE

$= \int_0^2 9.8(1000)(8)(2x)(3 - x) dx$
 $= \int_0^2 9.8(1000)(8)(4 - 2x)(x + 1) dx$
 $= \int_1^3 9.8(1000)(8)(6 - 2x)(x) dx$