

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK****USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE****MULTIPLE CHOICE: CIRCLE THE CORRECT ANSWER**

SCORE: ____ / 3 POINTS

The average value of $f(x) = \frac{3-x}{\sqrt{x}}$ on $[1, 4]$ is

$$[a] \Rightarrow \frac{4}{9}$$

$$[b] \Rightarrow \frac{1}{3}$$

$$[c] \Rightarrow -\frac{3}{4}$$

$$[d] \Rightarrow -1\frac{1}{3}$$

$$[e] \Rightarrow \frac{2}{3}$$

$$[f] \Rightarrow \frac{5}{9}$$

Find the value of c guaranteed by the Integral Mean Value Theorem for $f(x) = 6x^2 + 7$ on $[-4, 1]$.

SCORE: ____ / 4 POINTS

$$6c^2 + 7 = \frac{1}{1 - -4} \int_{-4}^1 (6x^2 + 7) dx$$

$$= \frac{1}{5} (2x^3 + 7x) \Big|_{-4}^1$$

$$= \frac{1}{5} (2(1^3 - (-4)^3) + 7(1 - -4))$$

$$6c^2 + 7 = 33, 2$$

$$6c^2 = 26$$

$$c = \pm \sqrt{\frac{13}{3}}$$

$-\frac{1}{2}$ IF YOUR ANSWER
INCLUDED BOTH $\pm \sqrt{\frac{13}{3}}$

$$c = -\sqrt{\frac{13}{3}} \in [-4, 1]$$

Find the arc length function for the curve $y = \frac{1}{6}(4x^2 + 2)^{\frac{3}{2}}$ with starting point $(1, \sqrt{6})$.

SCORE: ____ / 6 POINTS

Simplify your answer COMPLETELY.

$$s(x) = \int_1^x \sqrt{1 + \left(\frac{1}{6} \cdot \frac{3}{2} (4t^2 + 2)^{\frac{1}{2}} \cdot 8t\right)^2} dt$$

$$= \int_1^x \sqrt{1 + (2t \sqrt{4t^2 + 2})^2} dt$$

$$= \int_1^x \sqrt{1 + 4t^2(4t^2 + 2)} dt$$

$$= \int_1^x \sqrt{1 + 8t^2 + 16t^4} dt$$

$$= \int_1^x (1 + 4t^2) dt$$

$$= \left(t + \frac{4}{3}t^3\right) \Big|_1^x$$

$$= \frac{4}{3}x^3 + x - \frac{7}{3}$$

The curve $x = 4\sqrt{y-4}$ for $y \in [8, 13]$ is revolved around the y -axis.

SCORE: ___ / 10 POINTS

[a] Find the resulting surface area using a dy integral.

$$\begin{aligned}
 & \int_8^{13} 2\pi \left(4\sqrt{y-4} \right) \sqrt{1 + \left(\frac{4}{2\sqrt{y-4}} \right)^2} dy \\
 &= \int_8^{13} 8\pi \sqrt{y-4} \sqrt{1 + \frac{4}{y-4}} dy \\
 &= \int_8^{13} 8\pi \sqrt{y-4+4} dy \\
 &= \int_8^{13} 8\pi \sqrt{y} dy = 8\pi \left(\frac{2}{3} y^{\frac{3}{2}} \right) \Big|_8^{13} = \frac{16}{3} \pi (13^{\frac{3}{2}} - 8^{\frac{3}{2}})
 \end{aligned}$$

[b] Write, **BUT DO NOT EVALUATE**, a dx integral for the same surface area.

$$\begin{aligned}
 x &= 4\sqrt{y-4} \rightarrow y = 4 + \frac{x^2}{16} \\
 y &= 8 \rightarrow x = 8 \\
 y &= 13 \rightarrow x = 12
 \end{aligned}$$

$$\int_8^{12} 2\pi x \sqrt{1 + \left(\frac{2x}{16} \right)^2} dx = \int_8^{12} 2\pi x \sqrt{1 + \frac{x^2}{64}} dx$$

A vertical plate is partially submerged in water as shown on the right. Find the hydrostatic force on the plate.

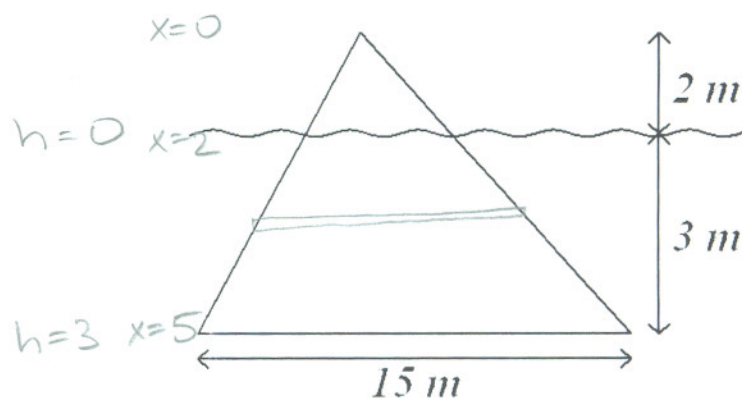
SCORE: ___ / 7 POINTS

NOTE: You MAY use the symbols ρ , δ and/or g in your final answers, if you write down their values underneath your answer.

$$x - h = 2 \rightarrow h = x - 2$$



$$\frac{w}{x_2^*} = \frac{15}{5} \rightarrow w = 3x_2^*$$



$$\begin{aligned}
 & \int_2^5 (x-2) \rho g (3x) dx \\
 &= 3\rho g \int_2^5 (x^2 - 2x) dx
 \end{aligned}$$

$$= 3\rho g \left(\frac{1}{3} x^3 - x^2 \right) \Big|_2^5$$

$$= 3\rho g \left(\frac{1}{3} (5^3 - 2^3) - (5^2 - 2^2) \right)$$

$$= 3\rho g (39 - 21)$$

$$= 3\rho g (18)$$

$$= 54\rho g \text{ N}$$

$$\rho = 1000, g = 9.8$$

What month is your birthday ?

What are the first 2 digits of your address ?

What are the last 2 digits of your zip code ?

What are the last 2 digits of your DeAnza ID number ?

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SCORE: ____ / 7 POINTS

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TOP

$$\frac{y}{2} = \frac{w}{15} \rightarrow y = 6$$

$$\frac{w-b}{x_i^*} = \frac{15}{5} \rightarrow y = 3x_i^* + 6$$

$$\int_0^3 x \rho g (3x+6) dx$$

$$= \rho g \int_0^3 (3x^2 + 6x) dx$$

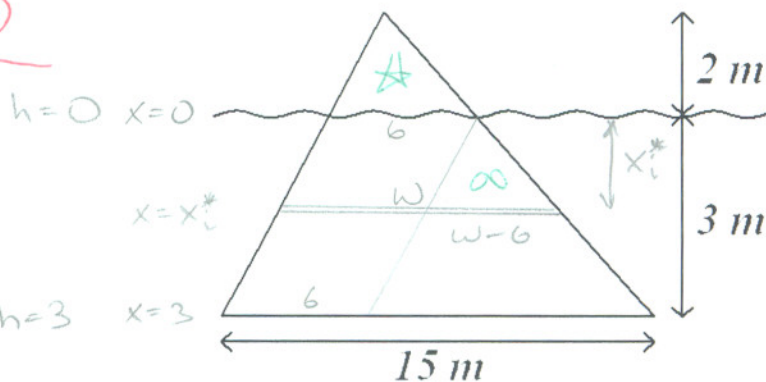
$$= \rho g \left(x^3 + 3x^2 \right) \Big|_0^3$$

$$= \rho g (13^3 + 3(3)^2)$$

$$= \rho g (27 + 27)$$

$$= 54 \rho g \text{ N}$$

$$\rho = 1000, g = 9.8$$



TOP

$$\frac{y}{2} = \frac{w}{15} \rightarrow y = 6$$

$$\frac{w-b}{3-x_i^*} = \frac{15}{5} \rightarrow w = 15 - 3x_i^*$$

$$\int_0^3 (3-x) \rho g (15-3x) dx$$

$$= \rho g \int_0^3 (45 - 24x + 3x^2) dx$$

$$= \rho g \left(45x - 12x^2 + x^3 \right) \Big|_0^3$$

$$= \rho g (45(3) - 12(3)^2 + (3)^3)$$

$$= \rho g (135 - 108 + 27)$$

$$= 54 \rho g \text{ N}$$

$$\rho = 1000, g = 9.8$$

$$h+x=3 \rightarrow h=3-x$$

$$h=0 \quad x=3$$

$$x=x_i^*$$

$$h=3 \quad x=0$$

