Find $\int \sec^2\sqrt{x}\,dx$ by first making a substitution, then using integration by parts.

SCORE: ___ / 6 POINTS

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}\,dx$

$dx=2\sqrt{x}\,du=2u\,du$

$2\ \underbrace{\int \sec^2 u\cdot 2u\,du}$ BY PARTS

u		dv
$2u$	$+$	$\sec^2 u$
2	$-$	$\tan u$
0		$\ln\lvert\sec u\rvert$

$= \underbrace{2u\tan u}_{1}\ \underbrace{-2\ln|\sec u|}_{1\frac{1}{2}}+C$

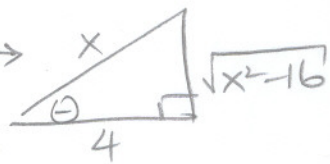
$= \underbrace{2\sqrt{x}\tan\sqrt{x}-2\ln|\sec\sqrt{x}|}_{1}\ \underbrace{+C}_{\frac{1}{2}}$

Find $\int \frac{\sqrt{x^2-16}}{x^2}\,dx$.

SCORE: ___ / 7 POINTS

$1\ \underbrace{x=4\sec\theta}$

$dx=4\sec\theta\tan\theta\,d\theta$



$\int \underbrace{\frac{4\tan\theta}{16\sec^2\theta}}_{1}\ \underbrace{4\sec\theta\tan\theta\,d\theta}_{1}$

$=\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$

$=\int \frac{\sec^2\theta-1}{\sec\theta}\,d\theta$

$= \underbrace{\int(\sec\theta-\cos\theta)}_{\frac{1}{2}}\,d\theta$

$= \underbrace{\ln|\sec\theta+\tan\theta|}_{1}\ \underbrace{-\sin\theta}_{\frac{1}{2}}+C$

$= \underbrace{\ln\left|\frac{x}{4}+\frac{\sqrt{x^2-16}}{4}\right|-\frac{\sqrt{x^2-16}}{4}}_{1}+C$

$= \underbrace{\ln\left|x+\sqrt{x^2-16}\right|-\frac{\sqrt{x^2-16}}{4}}_{\frac{1}{2}}\ \underbrace{+C}_{\frac{1}{2}}$

Find $\int \sec^3 x \tan^5 x \, dx$. **SCORE: ___ / 5 POINTS**

$1 \quad \underline{u = \sec x}$

$du = \sec x \tan x \, dx$

$\int \sec^2 x \tan^4 x \cdot \sec x \tan x \, dx$

$= \underbrace{\int u^2 (u^2 - 1)^2 \, du}_{1}$

$= \underbrace{\int (u^6 - 2u^4 + u^2) \, du}_{\frac{1}{2}}$

$= \underbrace{\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3}_{1} + C$

$= \underbrace{\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x}_{1} + \underbrace{C}_{\frac{1}{2}}$

Find $\int \frac{(\ln x)^2}{x^3} \, dx$. **SCORE: ___ / 5 POINTS**

u	dv
$(\ln x)^2$	x^{-3}
$+$	
$\frac{2\ln x}{x}$	$-\frac{1}{2}x^{-2}$
$2\ln x$	$-\frac{1}{2}x^{-3}$
$-$	
$\frac{2}{x}$	$\frac{1}{4}x^{-2}$
2	$\frac{1}{4}x^{-3}$
$+$	
0	$-\frac{1}{8}x^{-2}$

$\underbrace{-\frac{1}{2}x^{-2}(\ln x)^2}_{1\frac{1}{2}} \underbrace{-\frac{1}{2}x^{-2}\ln x}_{1\frac{1}{2}} \underbrace{-\frac{1}{4}x^{-2}}_{1\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$

Find $\int e^{-2x} \sin 4x \, dx$. **SCORE: ___ / 5 POINTS**

u	dv
$\sin 4x$	e^{-2x}
$+$	
$4\cos 4x$	$-\frac{1}{2}e^{-2x}$
$-$	
$-16\sin 4x$ $\pm$	$\frac{1}{4}e^{-2x}$

$\int e^{-2x} \sin 4x \, dx = \underbrace{-\frac{1}{2}e^{-2x}\sin 4x}_{1} - \underbrace{e^{-2x}\cos 4x}_{1}$

$\underbrace{-4\int e^{-2x}\sin 4x \, dx}_{1}$

$\underbrace{5\int e^{-2x}\sin 4x \, dx = -\frac{1}{2}e^{-2x}\sin 4x - e^{-2x}\cos 4x}_{\frac{1}{2}}$

$\int e^{-2x}\sin 4x \, dx = \underbrace{-\frac{1}{10}e^{-2x}\sin 4x - \frac{1}{5}e^{-2x}\cos 4x}_{1} + \underbrace{C}_{\frac{1}{2}}$

OR

Find $\int e^{-2x} \sin 4x \, dx$.

SCORE: ___ / 5 POINTS

u		dv
e^{-2x}	$+$	$\sin 4x$
$-2e^{-2x}$	$-$	$-\frac{1}{4}\cos 4x$
$4e^{-2x}$	$\pm$	$-\frac{1}{16}\sin 4x$

$$\int e^{-2x}\sin 4x\,dx = \underbrace{-\tfrac{1}{4}e^{-2x}\cos 4x}_{1}\,\underbrace{-\tfrac{1}{8}e^{-2x}\sin 4x}_{1}\,\underbrace{-\tfrac{1}{4}\int e^{-2x}\sin 4x\,dx}_{1}$$

$$\underbrace{\tfrac{5}{4}\int e^{-2x}\sin 4x\,dx = -\tfrac{1}{4}e^{-2x}\cos 4x - \tfrac{1}{8}e^{-2x}\sin 4x}_{\frac{1}{2}}$$

$$\int e^{-2x}\sin 4x\,dx = \underbrace{-\tfrac{1}{5}e^{-2x}\cos 4x - \tfrac{1}{10}e^{-2x}\sin 4x}_{1}\underbrace{+C}_{\frac{1}{2}}$$