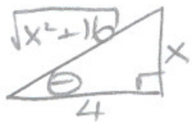
Find $\int \sec^2 \sqrt{x}\, dx$ by first making a substitution, then using integration by parts. **SCORE: ___ / 6 POINTS**

SEE 7:30 VERSION I

Find $\int \frac{1}{x^2\sqrt{x^2+16}}\, dx$. **SCORE: ___ / 7 POINTS**

1 $x = 4\tan\theta$

$dx = 4\sec^2\theta\, d\theta$



$\int \frac{1}{16\tan^2\theta \cdot 4\sec\theta} \, 4\sec^2\theta\, d\theta$

1 1

$= \frac{1}{16}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$

$= \frac{1}{16}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\, d\theta$

$= \frac{1}{16}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$ $\frac{1}{2}$

$= \frac{1}{16}\int \csc\theta\cot\theta\, d\theta$ $\frac{1}{2}$

$= -\frac{1}{16}\csc\theta + C$

$= -\frac{1}{16}\frac{\sqrt{x^2+16}}{x} + C$

$= -\frac{\sqrt{x^2+16}}{16x} + C$

$1\frac{1}{2}$ $\frac{1}{2}$

OR

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\frac{1}{16}\int \frac{1}{u^2}\, du = \frac{1}{16}\left(-\frac{1}{u}\right) + C$

$\frac{1}{2}$

$= -\frac{1}{16\sin\theta} + C$

1

Find $\int \sec^6 x \tan^4 x \, dx$. **SCORE: ___ / 5 POINTS**

SEE ADDITIONAL SOLUTION AT END

1 $u = \tan x$

$du = \sec^2 x \, dx$

$\int \sec^4 x \tan^4 x \cdot \sec^2 x \, dx$

$= \int (u^2+1)^2 u^4 \, du$ 1

$= \int (u^8 + 2u^6 + u^4) \, du$ $\frac{1}{2}$

$= \frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5 + C$ 1

$= \frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$ 1 $\frac{1}{2}$

Find $\int \frac{(\ln x)^2}{\sqrt{x}} \, dx$. **SCORE: ___ / 5 POINTS**

u		dv
$(\ln x)^2$		$x^{-\frac{1}{2}}$
	$+$	
$\frac{2\ln x}{x}$		$2x^{\frac{1}{2}}$
$2\ln x$		$2x^{-\frac{1}{2}}$
	$-$	
$\frac{2}{x}$		$4x^{\frac{1}{2}}$
2		$4x^{-\frac{1}{2}}$
	$+$	
0		$8x^{\frac{1}{2}}$

$$\underbrace{2x^{\frac{1}{2}}(\ln x)^2}_{1\frac{1}{2}} \underbrace{- 8x^{\frac{1}{2}}\ln x}_{1\frac{1}{2}} \underbrace{+ 16x^{\frac{1}{2}}}_{1\frac{1}{2}} \underbrace{+ C}_{\frac{1}{2}}$$

Find $\int e^{-4x} \cos 2x \, dx$. **SCORE: ___ / 5 POINTS**

u		dv
$\cos 2x$		e^{-4x}
	$+$	
$-2\sin 2x$		$-\frac{1}{4}e^{-4x}$
	$-$	
$-4\cos 2x$	$+$	$\frac{1}{16}e^{-4x}$

$\int e^{-4x}\cos 2x \, dx = \underbrace{-\frac{1}{4}e^{-4x}\cos 2x}_{1} + \underbrace{\frac{1}{8}e^{-4x}\sin 2x}_{1}$

$\underbrace{-\frac{1}{4}\int e^{-4x}\cos 2x \, dx}_{1}$

$\underbrace{\frac{5}{4}\int e^{-4x}\cos 2x \, dx = -\frac{1}{4}e^{-4x}\cos 2x + \frac{1}{8}e^{-4x}\sin 2x}_{\frac{1}{2}}$

$\int e^{-4x}\cos 2x \, dx = \underbrace{-\frac{1}{5}e^{-4x}\cos 2x + \frac{1}{10}e^{-4x}\sin 2x}_{1} \underbrace{+ C}_{\frac{1}{2}}$

OR

Find $\int e^{-4x}\cos 2x\,dx$.

SCORE: ___ / 5 POINTS

u		dv
e^{-4x}	$+$	$\cos 2x$
$-4e^{-4x}$	$-$	$\frac{1}{2}\sin 2x$
$16e^{-4x}$	$+$	$-\frac{1}{4}\cos 2x$

$$\int e^{-4x}\cos 2x\,dx = \underbrace{\tfrac{1}{2}e^{-4x}\sin 2x}_{1}\ \underbrace{- e^{-4x}\cos 2x}_{1}$$

$$\underbrace{-4\int e^{-4x}\cos 2x\,dx}_{1}$$

$$\underbrace{5\int e^{-4x}\cos 2x\,dx = \tfrac{1}{2}e^{-4x}\sin 2x - e^{-4x}\cos 2x}_{\frac{1}{2}}$$

$$\int e^{-4x}\cos 2x\,dx = \underbrace{\tfrac{1}{10}e^{-4x}\sin 2x - \tfrac{1}{5}e^{-4x}\cos 2x}_{1} + \underbrace{C}_{\frac{1}{2}}$$

THIS SOLUTION IS HIGHLY NOT RECOMMENDED (COMPARE TO OTHER SOLUTION)

Find $\int \sec^6 x \tan^4 x \, dx$.

SCORE: ___ / 5 POINTS

$= \int (\sec^6 x)(\sec^2 x - 1)^2 \, dx$

$= \int (\sec^6 x)(\sec^4 x - 2\sec^2 x + 1)^2 \, dx$

$= \int (\sec^{10} x - 2\sec^8 x + \sec^6 x) \, dx$ $\frac{1}{2}$

$= \int \sec^{10} x \, dx - 2\int \sec^8 x \, dx + \int \sec^6 x \, dx$

$= \frac{1}{9}\sec^8 x \tan x + \frac{8}{9}\int \sec^8 x \, dx - 2\int \sec^8 x \, dx + \int \sec^6 x \, dx$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{9}\int \sec^8 x \, dx + \int \sec^6 x \, dx$ $\frac{1}{2}$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{9}\left[\frac{1}{7}\sec^6 x \tan x + \frac{6}{7}\int \sec^6 x \, dx\right] + \int \sec^6 x \, dx$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x - \frac{20}{21}\int \sec^6 x \, dx + \int \sec^6 x \, dx$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{21}\int \sec^6 x \, dx$ 1

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{21}\left[\frac{1}{5}\sec^4 x \tan x + \frac{4}{5}\int \sec^4 x \, dx\right]$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{105}\sec^4 x \tan x + \frac{4}{105}\int \sec^4 x \, dx$ 1

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{105}\sec^4 x \tan x + \frac{4}{105}\left[\frac{1}{3}\sec^2 x \tan x + \frac{2}{3}\int \sec^2 x \, dx\right]$

$= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{105}\sec^4 x \tan x + \frac{4}{315}\sec^2 x \tan x + \frac{8}{315}\int \sec^2 x \, dx$ 1

$$\underbrace{= \frac{1}{9}\sec^8 x \tan x - \frac{10}{63}\sec^6 x \tan x + \frac{1}{105}\sec^4 x \tan x + \frac{4}{315}\sec^2 x \tan x + \frac{8}{315}\tan x}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$