Math 1B (7:30am – 8:20am)
Quiz 8 Version C
Fri Jun 10, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du \quad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du \quad (n \neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \frac{5x^2+10}{x^3-2x^2+10x}\,dx$. **SCORE: ___ / 7 POINTS**

$$\frac{5x^2+10}{x^3-2x^2+10x} = \frac{5x^2+10}{x((x-1)^2+9)} = \underbrace{\frac{A}{x} + \frac{B(2x-2)+C(3)}{((x-1)^2+9)}}_{1} = \frac{A((x-1)^2+9)+Bx(2x-2)+C(3x)}{x((x-1)^2+9)}$$

$\underbrace{5x^2+10 = A((x-1)^2+9)+Bx(2x-2)+C(3x)}_{\frac{1}{2}}$

$x=0:\quad 10 = 10A \quad\Rightarrow\quad \underbrace{A=1}_{1}$

$x=1:\quad 15 = 9A+3C \quad\Rightarrow\quad \underbrace{C=2}_{1}$

$x=-1:\quad 15 = 13A+4B-3C \quad\Rightarrow\quad \underbrace{B=2}_{1}$

$\frac{1}{2}$

SANITY CHECK: $x=2:\quad \frac{5x^2+10}{x^3-2x^2+10x} = \frac{30}{20} = \frac{3}{2} \qquad \frac{1}{x} + \frac{2(2x-2)+2(3)}{((x-1)^2+9)} = \frac{1}{2} + \frac{10}{10} = \frac{3}{2}$

$$\int\left(\frac{1}{x} + \frac{2(2x-2)+2(3)}{((x-1)^2+9)}\right)dx = \underbrace{\ln|x|}_{\frac{1}{2}} + \underbrace{2\ln(x^2-2x+10)}_{\frac{1}{2}} + \underbrace{2\tan^{-1}\frac{x-1}{3}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

Find $\int \cos(\sqrt[3]{x})\,dx$. **SCORE: ___ / 6 POINTS**

Let $\underbrace{x = u^3 \quad OR \quad u = \sqrt[3]{x}}_{1} \quad\Rightarrow\quad dx = 3u^2\,du$

$\int \cos(\sqrt[3]{x})\,dx = \underbrace{\int 3u^2\cos u\,du}_{1}$

1 IF 2 TERMS CORRECT
2 ALL 3

f		dg
$3u^2$	+	$\cos u$
$6u$	–	$\sin u$
6	+	$-\cos u$
0		$-\sin u$

$\int 3u^2\cos u\,du = 3u^2\sin u + 6u\cos u - 6\sin u + C$

$$\int \cos(\sqrt[3]{x})\,dx = \underbrace{3x^{\frac{2}{3}}\sin x^{\frac{1}{3}}}_{\frac{1}{2}} + \underbrace{6x^{\frac{1}{3}}\cos x^{\frac{1}{3}}}_{\frac{1}{2}} \underbrace{- 6\sin x^{\frac{1}{3}}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

Find $\int \frac{18-x}{x^3-6x^2+9x}\,dx$. **SCORE: ___ / 7 POINTS**

$$\frac{18-x}{x^3-6x^2+9x} = \frac{18-x}{x(x-3)^2} = \underbrace{\boxed{\frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}}}_{1} = \frac{A(x-3)^2+Bx(x-3)+Cx}{x(x-3)^2}$$

$$\underbrace{18-x = A(x-3)^2+Bx(x-3)+Cx}_{\frac{1}{2}}$$

$$x=0: \quad 18 = 9A \quad \Rightarrow \quad \underbrace{A=2}_{1}$$

$$x=3: \quad 15 = 3C \quad \Rightarrow \quad \underbrace{C=5}_{1}$$

$$x=1: \quad 17 = 4A-2B+C \quad \Rightarrow \quad \underbrace{B=-2}_{1}$$

$\frac{1}{2}$

SANITY CHECK: $\quad x=2: \quad \frac{18-x}{x^3-6x^2+9x} = \frac{16}{2} = 8 \qquad \frac{2}{x}+\frac{-2}{x-3}+\frac{5}{(x-3)^2} = \frac{2}{2}+\frac{-2}{-1}+\frac{5}{1} = 8$

$$\int\left(\frac{2}{x}+\frac{-2}{x-3}+\frac{5}{(x-3)^2}\right)dx = 2\ln|x| - 2\ln|x-3| - \frac{5}{x-3} + C$$

Find $\int \frac{1}{x+x\sqrt{x}}\,dx$. **SCORE: ___ / 7 POINTS**

Let $\overbrace{\underbrace{x=(u-1)^2 \quad OR \quad u = 1+\sqrt{x} \Rightarrow}}^{\frac{1}{2}} \quad dx = 2(u-1)\,du$

$$\int \frac{1}{x+x\sqrt{x}}\,dx = \int \frac{1}{x(1+\sqrt{x})}\,dx = \int \frac{2(u-1)}{(u-1)^2 u}\,du = \underbrace{\int \frac{2}{(u-1)u}\,du}_{1}$$

SEE ALTERNATE SOLUTION 7:30 VERSIONS

$$\frac{2}{(u-1)u} = \underbrace{\boxed{\frac{A}{u-1}+\frac{B}{u}}}_{\frac{1}{2}} = \frac{Au+B(u-1)}{(u-1)u}$$

$$\underbrace{2 = Au+B(u-1)}_{\frac{1}{2}}$$

$$u=0: \quad 2 = -B \quad \Rightarrow \underbrace{B=-2}_{\frac{1}{2}}$$

$$u=1: \quad \underbrace{2=A}_{\frac{1}{2}}$$

$\frac{1}{2}$

SANITY CHECK: $\quad u=2: \quad \frac{2}{(u-1)u} = \frac{2}{2} = 1 \qquad \frac{2}{u-1}+\frac{-2}{u} = \frac{2}{1}+\frac{-2}{2} = 1$

$$\int\left(\frac{2}{u-1}+\frac{-2}{u}\right)du = \underbrace{2\ln|u-1|}_{\frac{1}{2}}\underbrace{-2\ln|u|}_{\frac{1}{2}}+C = \underbrace{2\ln\sqrt{x}-2\ln(1+\sqrt{x})}_{1}+C = \underbrace{\ln x - 2\ln(1+\sqrt{x})}_{\frac{1}{2}}\underbrace{+C}_{\frac{1}{2}}$$

Find $\int_{-\pi}^{\pi} x^7 \cos x^3\,dx$. **SCORE: ___ / 3 POINTS**

$$1 \quad \underbrace{(-x)^7\cos(-x)^3 = -x^7\cos(-x^3) = -x^7\cos x^3}$$

Since the integrand $\underbrace{\text{is odd}}_{1}$ and continuous, and the interval is symmetric, $\int_{-\pi}^{\pi} x^7\cos x^3\,\underbrace{dx = 0}_{1}$.