Math 1B (7:30am – 8:20am)
Quiz 8 Version D
Fri Jun 10, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du \qquad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du \qquad (n \neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \frac{18-x}{x^3-6x^2+9x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{18-x}{x^3-6x^2+9x} = \frac{18-x}{x(x-3)^2} = \left[\frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}\right] = \frac{A(x-3)^2+Bx(x-3)+Cx}{x(x-3)^2}$$

$18-x = A(x-3)^2+Bx(x-3)+Cx$ ½

$x=0:\ 18=9A \Rightarrow A=2$ 1

$x=3:\ 15=3C \Rightarrow C=5$ 1

$x=1:\ 17=4A-2B+C \Rightarrow B=-2$ 1

SANITY CHECK: $x=2:\ \frac{18-x}{x^3-6x^2+9x}=\frac{16}{2}=8 \qquad \frac{2}{x}+\frac{-2}{x-3}+\frac{5}{(x-3)^2}=\frac{2}{2}+\frac{-2}{-1}+\frac{5}{1}=8$ ½

$$\int\left(\frac{2}{x}+\frac{-2}{x-3}+\frac{5}{(x-3)^2}\right)dx = 2\ln|x| - 2\ln|x-3| - \frac{5}{x-3} + C$$

½ ½ ½ ½

Find $\int \frac{5x^2+10}{x^3-2x^2+10x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{5x^2+10}{x^3-2x^2+10x} = \frac{5x^2+10}{x((x-1)^2+9)} = \left[\frac{A}{x}+\frac{B(2x-2)+C(3)}{((x-1)^2+9)}\right] = \frac{A((x-1)^2+9)+Bx(2x-2)+C(3x)}{x((x-1)^2+9)}$$

1

$5x^2+10 = A((x-1)^2+9)+Bx(2x-2)+C(3x)$ ½

$x=0:\ 10=10A \Rightarrow A=1$ 1

$x=1:\ 15=9A+3C \Rightarrow C=2$ 1

$x=-1:\ 15=13A+4B-3C \Rightarrow B=2$ 1

SANITY CHECK: $x=2:\ \frac{5x^2+10}{x^3-2x^2+10x}=\frac{30}{20}=\frac{3}{2} \qquad \frac{1}{x}+\frac{2(2x-2)+2(3)}{((x-1)^2+9)}=\frac{1}{2}+\frac{10}{10}=\frac{3}{2}$ ½

$$\int\left(\frac{1}{x}+\frac{2(2x-2)+2(3)}{((x-1)^2+9)}\right)dx = \ln|x| + 2\ln(x^2-2x+10) + 2\tan^{-1}\frac{x-1}{3} + C$$

½ ½ ½ ½

Find $\int \cos(\sqrt[3]{x})\,dx$.

SCORE: ___ / 6 POINTS

Let $x = u^3$ OR $u = \sqrt[3]{x} \Rightarrow dx = 3u^2\,du$ (1)

$\int \cos(\sqrt[3]{x})\,dx = \int 3u^2 \cos u\,du$ (1)

f	dg	
$3u^2$	$\cos u$	+
$6u$	$\sin u$	−
6	$-\cos u$	+
0	$-\sin u$	

1 POINT IF 2 TERMS CORRECT
2 POINTS IF ALL 3 TERMS CORRECT

$\int 3u^2 \cos u\,du = 3u^2 \sin u + 6u\cos u - 6\sin u + C$

$\int \cos(\sqrt[3]{x})\,dx = 3x^{\frac{2}{3}} \sin x^{\frac{1}{3}} + 6x^{\frac{1}{3}} \cos x^{\frac{1}{3}} - 6\sin x^{\frac{1}{3}} + C$ ($\frac{1}{2}$ each)

Find $\int \frac{1}{x + x\sqrt{x}}\,dx$.

SCORE: ___ / 7 POINTS

SEE ALTERNATE SOLUTION 7:30 VERSION I

Let $x = u^2$ OR $u = \sqrt{x} \Rightarrow dx = 2u\,du$ ($\frac{1}{2}$)

$\int \frac{1}{x + x\sqrt{x}}\,dx = \int \frac{2u}{u^2 + u^3}\,du = \int \frac{2}{u + u^2}\,du$ (1)

$\frac{2}{u+u^2} = \frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u} = \frac{A(1+u)+Bu}{u(1+u)}$ ($\frac{1}{2}$)

$2 = A(1+u) + Bu$ ($\frac{1}{2}$)

$u = 0$: $2 = A$ ($\frac{1}{2}$)

$u = -1$: $2 = -B \Rightarrow B = -2$ ($\frac{1}{2}$)

SANITY CHECK: $u = 2$: $\frac{2}{u+u^2} = \frac{2}{6} = \frac{1}{3}$ ($\frac{1}{2}$) $\quad \frac{2}{u} + \frac{-2}{1+u} = \frac{2}{2} + \frac{-2}{3} = \frac{1}{3}$

$\int \left(\frac{2}{u} + \frac{-2}{1+u}\right) du = 2\ln|u| - 2\ln|1+u| + C = 2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C = \ln x - 2\ln(1+\sqrt{x}) + C$

($\frac{1}{2}$ each) ($\frac{1}{2}$ each) (1) ($\frac{1}{2}$) ($\frac{1}{2}$)

Find $\int_{-\pi}^{\pi} x^7 \cos x^3\,dx$.

SCORE: ___ / 3 POINTS

(1) $(-x)^7 \cos(-x)^3 = -x^7 \cos(-x^3) = -x^7 \cos x^3$

Since the integrand is odd and continuous, and the interval is symmetric, $\int_{-\pi}^{\pi} x^7 \cos x^3\,dx = 0$.

(1) (1)