Math 1B (7:30am – 8:20am)
Quiz 8 Version S
Fri Jun 10, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du \quad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du \quad (n \neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \cos(\sqrt[3]{x})\,dx$. **SCORE: ___ / 6 POINTS**

Let $x = u^3$ OR $u = \sqrt[3]{x} \Rightarrow dx = 3u^2\,du$ 1

$\int \cos(\sqrt[3]{x})\,dx = \int 3u^2 \cos u\,du$ 1

f	dg
$3u^2$ (+)	$\cos u$
$6u$ (−)	$\sin u$
6 (+)	$-\cos u$
0	$-\sin u$

$\int 3u^2\cos u\,du = \underbrace{3u^2\sin u + 6u\cos u - 6\sin u}_{} + C$

1 IF 2 TERMS CORRECT
2 ALL 3

$$\int \cos(\sqrt[3]{x})\,dx = \underbrace{3x^{\frac{2}{3}}\sin x^{\frac{1}{3}}}_{\frac{1}{2}} + \underbrace{6x^{\frac{1}{3}}\cos x^{\frac{1}{3}}}_{\frac{1}{2}} - \underbrace{6\sin x^{\frac{1}{3}}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

Find $\int \frac{18-x}{x^3-6x^2+9x}\,dx$. **SCORE: ___ / 7 POINTS**

$$\frac{18-x}{x^3-6x^2+9x} = \frac{18-x}{x(x-3)^2} = \boxed{\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}}\ 1 = \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$$

$18 - x = A(x-3)^2 + Bx(x-3) + Cx$ $\frac{1}{2}$

$x = 0$: $18 = 9A \Rightarrow A = 2$ 1

$x = 3$: $15 = 3C \Rightarrow C = 5$ 1

$x = 1$: $17 = 4A - 2B + C \Rightarrow B = -2$ 1

SANITY CHECK: $x = 2$: $\frac{18-x}{x^3-6x^2+9x} = \frac{16}{2} = 8$ $\quad \frac{2}{x} + \frac{-2}{x-3} + \frac{5}{(x-3)^2} = \frac{2}{2} + \frac{-2}{-1} + \frac{5}{1} = 8$ $\frac{1}{2}$

$$\int\left(\frac{2}{x} + \frac{-2}{x-3} + \frac{5}{(x-3)^2}\right)dx = \underbrace{2\ln|x|}_{\frac{1}{2}} - \underbrace{2\ln|x-3|}_{\frac{1}{2}} - \underbrace{\frac{5}{x-3}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

Find $\int \frac{1}{x+x\sqrt{x}}\,dx$. SCORE: ___ / 7 POINTS

SEE ALTERNATE SOLUTION 7:30 VERSION C

$$\text{Let } \overset{\frac{1}{2}}{\underbrace{x=u^2 \quad OR \quad u=\sqrt{x}}} \Rightarrow \quad dx=2u\,du$$

$$\int \frac{1}{x+x\sqrt{x}}\,dx \;=\; \int \frac{2u}{u^2+u^3}\,du \;=\; \underbrace{\int \frac{2}{u+u^2}\,du}_{1}$$

$$\frac{2}{u+u^2} \;=\; \frac{2}{u(1+u)} \;=\; \underbrace{\boxed{\frac{A}{u}+\frac{B}{1+u}}}_{\frac{1}{2}} \;=\; \frac{A(1+u)+Bu}{u(1+u)}$$

$$\tfrac{1}{2}\quad \underbrace{2=A(1+u)+Bu}$$

$$u=0: \quad \underbrace{2=A}_{\frac{1}{2}}$$

$$u=-1: \quad 2=-B \quad \Rightarrow \quad \underbrace{B=-2}_{\frac{1}{2}}$$

SANITY CHECK: $\quad u=2: \quad \frac{2}{u+u^2}=\frac{2}{6}=\frac{1}{3} \qquad \frac{2}{u}+\frac{-2}{1+u}=\frac{2}{2}+\frac{-2}{3}=\frac{1}{3}$ $\quad \frac{1}{2}$

$$\int\left(\frac{2}{u}+\frac{-2}{1+u}\right)du \;=\; \underbrace{2\ln|u|}_{\frac{1}{2}}\underbrace{-2\ln|1+u|}_{\frac{1}{2}}+C \;=\; \underbrace{2\ln\sqrt{x}-2\ln(1+\sqrt{x})}_{1}+C \;=\; \underbrace{\ln x-2\ln(1+\sqrt{x})}_{\frac{1}{2}}\underbrace{+C}_{\frac{1}{2}}$$

Find $\int_{-\pi}^{\pi} x^7\cos x^3\,dx$. SCORE: ___ / 3 POINTS

$$1\quad \underbrace{(-x)^7\cos(-x)^3 \;=\; -x^7\cos(-x^3) \;=\; -x^7\cos x^3}$$

Since the integrand $\underbrace{\text{is odd}}_{1}$ and continuous, and the interval is symmetric, $\int_{-\pi}^{\pi} x^7\cos x^3\,dx \underbrace{=0}_{1}$.

Find $\int \frac{5x^2+10}{x^3-2x^2+10x}\,dx$. SCORE: ___ / 7 POINTS

$$\frac{5x^2+10}{x^3-2x^2+10x} \;=\; \frac{5x^2+10}{x((x-1)^2+9)} \;=\; \underbrace{\boxed{\frac{A}{x}+\frac{B(2x-2)+C(3)}{((x-1)^2+9)}}}_{1} \;=\; \frac{A((x-1)^2+9)+Bx(2x-2)+C(3x)}{x((x-1)^2+9)}$$

$$\underbrace{5x^2+10=A((x-1)^2+9)+Bx(2x-2)+C(3x)}_{\frac{1}{2}}$$

$$x=0: \quad 10=10A \quad \Rightarrow \quad \underbrace{A=1}_{1}$$

$$x=1: \quad 15=9A+3C \quad \Rightarrow \quad \underbrace{C=2}_{1}$$

$$x=-1: \quad 15=13A+4B-3C \quad \Rightarrow \quad \underbrace{B=2}_{1}$$

SANITY CHECK: $\quad x=2: \quad \frac{5x^2+10}{x^3-2x^2+10x}=\frac{30}{20}=\frac{3}{2} \qquad \frac{1}{x}+\frac{2(2x-2)+2(3)}{((x-1)^2+9)}=\frac{1}{2}+\frac{10}{10}=\frac{3}{2}$ $\quad \frac{1}{2}$

$$\int\left(\frac{1}{x}+\frac{2(2x-2)+2(3)}{((x-1)^2+9)}\right)dx \;=\; \underbrace{\ln|x|}_{\frac{1}{2}}+\underbrace{2\ln(x^2-2x+10)}_{\frac{1}{2}}+\underbrace{2\tan^{-1}\frac{x-1}{3}}_{\frac{1}{2}}\underbrace{+C}_{\frac{1}{2}}$$