Math 1B (9:30am – 10:20am)
Quiz 8 Version E
Fri Jun 10, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du \quad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du \quad (n \neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \frac{x+26}{x^3+4x^2+13x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{x+26}{x^3+4x^2+13x} = \frac{x+26}{x((x+2)^2+9)} = \frac{A}{x} + \frac{B(2x+4)+C(3)}{((x+2)^2+9)} = \frac{A((x+2)^2+9)+Bx(2x+4)+C(3x)}{x((x+2)^2+9)} \quad 1$$

$x+26 = A((x+2)^2+9)+Bx(2x+4)+C(3x)$ $\frac{1}{2}$

$x=0: \quad 26 = 13A \quad \Rightarrow \quad A=2$ 1

$x=-2: \quad 24 = 9A-6C \quad \Rightarrow \quad C=-1$ 1

$x=1: \quad 27 = 18A+6B+3C \quad \Rightarrow \quad B=-1$ 1

$\frac{1}{2}$

SANITY CHECK: $x=2: \quad \frac{x+26}{x^3+4x^2+13x} = \frac{28}{50} = \frac{14}{25} \qquad \frac{2}{x} + \frac{-(2x+4)-(3)}{((x+2)^2+9)} = \frac{2}{2} + \frac{-11}{25} = \frac{14}{25}$

$$\int\left(\frac{2}{x} + \frac{-(2x+4)-(3)}{((x+2)^2+9)}\right)dx = \underbrace{2\ln|x|}_{\frac{1}{2}} \underbrace{-\ln(x^2+4x+13)}_{\frac{1}{2}} \underbrace{-\tan^{-1}\frac{x+2}{3}}_{\frac{1}{2}} \underbrace{+C}_{\frac{1}{2}}$$

Find $\int (\arcsin x)^2\,dx$.

SCORE: ___ / 6 POINTS

Let $x = \sin u$ OR $u = \arcsin x \Rightarrow dx = \cos u\,du$ 1

$\int (\arcsin x)^2\,dx = \int u^2 \cos u\,du$ 1

1 IF 2 TERMS CORRECT
2 ALL 3

f		dg
u^2	$+$	$\cos u$
$2u$	$-$	$\sin u$
2	$+$	$-\cos u$
0		$-\sin u$

$\int u^2\cos u\,du = u^2\sin u + 2u\cos u - 2\sin u + C$

$$\int (\arcsin x)^2\,dx = \underbrace{x(\arcsin x)^2}_{\frac{1}{2}} \underbrace{+2\sqrt{1-x^2}\arcsin x}_{\frac{1}{2}} \underbrace{-2x}_{\frac{1}{2}} \underbrace{+C}_{\frac{1}{2}}$$

SEE ALTERNATE SOLUTION 9:30 VERSION R

Find $\int \frac{1}{x+x\sqrt{x}}\,dx$. SCORE: ___ / 7 POINTS

Let $\underbrace{x = u^2 \quad OR \quad u=\sqrt{x}}_{\frac{1}{2}} \Rightarrow \quad dx = 2u\,du$

$$\int \frac{1}{x+x\sqrt{x}}\,dx \;=\; \int \frac{2u}{u^2+u^3}\,du \;=\; \underbrace{\int \frac{2}{u+u^2}\,du}_{1}$$

$$\frac{2}{u+u^2} \;=\; \frac{2}{u(1+u)} \;=\; \underbrace{\frac{A}{u}+\frac{B}{1+u}}_{\frac{1}{2}} \;=\; \frac{A(1+u)+Bu}{u(1+u)}$$

$\underbrace{2 = A(1+u)+Bu}_{\frac{1}{2}}$

$u=0:\ \underbrace{2=A}_{\frac{1}{2}}$

$u=-1:\quad 2=-B \quad \Rightarrow \underbrace{B=-2}_{\frac{1}{2}}$

SEE ALTERNATE SOLUTION 9:30 VERSION R

SANITY CHECK: $u=2:\ \frac{2}{u+u^2} = \frac{2}{6} = \frac{1}{3} \qquad \frac{2}{u}+\frac{-2}{1+u} = \frac{2}{2}+\frac{-2}{3} = \frac{1}{3}$ $\frac{1}{2}$

$$\int\left(\frac{2}{u}+\frac{-2}{1+u}\right)du \;=\; \underbrace{2\ln|u|}_{\frac{1}{2}} \underbrace{-\,2\ln|1+u|}_{\frac{1}{2}}+C \;=\; \underbrace{2\ln\sqrt{x}-2\ln(1+\sqrt{x})}_{1}+C \;=\; \underbrace{\ln x - 2\ln(1+\sqrt{x})}_{\frac{1}{2}}\underbrace{+\,C}_{\frac{1}{2}}$$

Find $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx$. SCORE: ___ / 3 POINTS

$$\underbrace{\frac{(-x)^5}{\sqrt{1+(-x)^4}} \;=\; \frac{-x^5}{\sqrt{1+x^4}} \;=\; -\frac{x^5}{\sqrt{1+x^4}}}_{1}$$

Since the integrand is $\underbrace{\text{odd}}_{1}$ and continuous, and the interval is symmetric, $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx \underbrace{= 0}_{1}$.

Find $\int \frac{x^2+2}{x^3+2x^2+x}\,dx$. SCORE: ___ / 7 POINTS

$$\frac{x^2+2}{x^3+2x^2+x} \;=\; \frac{x^2+2}{x(x+1)^2} \;=\; \underbrace{\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}}_{1} \;=\; \frac{A(x+1)^2+Bx(x+1)+Cx}{x(x+1)^2}$$

$\underbrace{x^2+2 = A(x+1)^2+Bx(x+1)+Cx}_{\frac{1}{2}}$

$x=0:\ \underbrace{2=A}_{1}$

$x=-1:\quad 3=-C \quad \Rightarrow \underbrace{C=-3}_{1}$

$x=1:\quad 3=4A+2B+C \quad \Rightarrow \underbrace{B=-1}_{1}$

SANITY CHECK: $x=2:\ \frac{x^2+2}{x^3+2x^2+x} = \frac{6}{18} = \frac{1}{3} \qquad \frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2} = \frac{2}{2}+\frac{-1}{3}+\frac{-3}{9} = \frac{1}{3}$ $\frac{1}{2}$

$$\int\left(\frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2}\right)dx \;=\; \underbrace{2\ln|x|}_{\frac{1}{2}} \underbrace{-\,\ln|x+1|}_{\frac{1}{2}} + \underbrace{\frac{3}{x+1}}_{\frac{1}{2}} \underbrace{+\,C}_{\frac{1}{2}}$$