Math 1B (9:30am – 10:20am)
Quiz 8 Version O
Fri Jun 10, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du \qquad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du \qquad (n \neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \frac{x^2+2}{x^3+2x^2+x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{x^2+2}{x^3+2x^2+x} = \frac{x^2+2}{x(x+1)^2} = \left[\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right] = \frac{A(x+1)^2+Bx(x+1)+Cx}{x(x+1)^2}$$

$x^2+2 = A(x+1)^2+Bx(x+1)+Cx$ ½

$x=0$: $2=A$ 1

$x=-1$: $3=-C \Rightarrow C=-3$ 1

$x=1$: $3=4A+2B+C \Rightarrow B=-1$ 1

SANITY CHECK: $x=2$: $\frac{x^2+2}{x^3+2x^2+x}=\frac{6}{18}=\frac{1}{3}$ ½ $\frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2}=\frac{2}{2}+\frac{-1}{3}+\frac{-3}{9}=\frac{1}{3}$

$$\int\left(\frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2}\right)dx = 2\ln|x| - \ln|x+1| + \frac{3}{x+1} + C$$

½ ½ ½ ½

Find $\int \frac{x+26}{x^3+4x^2+13x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{x+26}{x^3+4x^2+13x} = \frac{x+26}{x((x+2)^2+9)} = \left[\frac{A}{x}+\frac{B(2x+4)+C(3)}{((x+2)^2+9)}\right] = \frac{A((x+2)^2+9)+Bx(2x+4)+C(3x)}{x((x+2)^2+9)}$$

$x+26 = A((x+2)^2+9)+Bx(2x+4)+C(3x)$ ½

$x=0$: $26=13A \Rightarrow A=2$ 1

$x=-2$: $24=9A-6C \Rightarrow C=-1$ 1

$x=1$: $27=18A+6B+3C \Rightarrow B=-1$ 1

SANITY CHECK: $x=2$: $\frac{x+26}{x^3+4x^2+13x}=\frac{28}{50}=\frac{14}{25}$ ½ $\frac{2}{x}+\frac{-(2x+4)-(3)}{((x+2)^2+9)}=\frac{2}{2}+\frac{-11}{25}=\frac{14}{25}$

$$\int\left(\frac{2}{x}+\frac{-(2x+4)-(3)}{((x+2)^2+9)}\right)dx = 2\ln|x| - \ln(x^2+4x+13) - \tan^{-1}\frac{x+2}{3} + C$$

½ ½ ½ ½

Find $\int (\arcsin x)^2\,dx$.

SCORE: ___ / 6 POINTS

Let $x = \sin u$ OR $u = \arcsin x \Rightarrow dx = \cos u\,du$ (1)

$\int (\arcsin x)^2\,dx = \int u^2 \cos u\,du$ (1)

1 POINT IF 2 TERMS CORRECT (2)
ALL 3

f	dg
u^2	$\cos u$
$2u$	$\sin u$
2	$-\cos u$
0	$-\sin u$

$\int u^2 \cos u\,du = u^2 \sin u + 2u\cos u - 2\sin u + C$

$\int (\arcsin x)^2\,dx = x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$ ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$)

SEE ALTERNATE SOLUTIONS 9:30 VERSION

Find $\int \frac{1}{x + x\sqrt{x}}\,dx$.

SCORE: ___ / 7 POINTS

Let $x = u^2$ OR $u = \sqrt{x} \Rightarrow dx = 2u\,du$ ($\frac{1}{2}$)

$\int \frac{1}{x + x\sqrt{x}}\,dx = \int \frac{2u}{u^2 + u^3}\,du = \int \frac{2}{u + u^2}\,du$ (1)

$\frac{2}{u+u^2} = \frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u} = \frac{A(1+u)+Bu}{u(1+u)}$ ($\frac{1}{2}$)

$2 = A(1+u) + Bu$ ($\frac{1}{2}$)

$u = 0$: $2 = A$ ($\frac{1}{2}$)

$u = -1$: $2 = -B \Rightarrow B = -2$ ($\frac{1}{2}$)

SANITY CHECK: $u = 2$: $\frac{2}{u+u^2} = \frac{2}{6} = \frac{1}{3}$ $\quad \frac{2}{u} + \frac{-2}{1+u} = \frac{2}{2} + \frac{-2}{3} = \frac{1}{3}$

$\int \left(\frac{2}{u} + \frac{-2}{1+u}\right) du = 2\ln|u| - 2\ln|1+u| + C = 2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C = \ln x - 2\ln(1+\sqrt{x}) + C$

($\frac{1}{2}$, $\frac{1}{2}$, 1, $\frac{1}{2}$, $\frac{1}{2}$)

Find $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx$.

SCORE: ___ / 3 POINTS

$\frac{(-x)^5}{\sqrt{1+(-x)^4}} = \frac{-x^5}{\sqrt{1+x^4}} = -\frac{x^5}{\sqrt{1+x^4}}$ (1)

Since the integrand is odd and continuous, and the interval is symmetric, $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx = 0$. (1, 1)