Math 1B (9:30am – 10:20am)
Quiz 8 Version R
Fri Jun 10, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du \qquad (n\neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du \qquad (n\neq 1)$$

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK
USE PROPER NOTATION & SIMPLIFY ALL ANSWERS WHERE REASONABLE

Find $\int \frac{1}{x+x\sqrt{x}}\,dx$.

SCORE: ___ / 7 POINTS

Let $x=(u-1)^2$ OR $u=1+\sqrt{x} \Rightarrow dx = 2(u-1)\,du$ (½)

$$\int \frac{1}{x+x\sqrt{x}}\,dx = \int \frac{1}{x(1+\sqrt{x})}\,dx = \int \frac{2(u-1)}{(u-1)^2 u}\,du = \int \frac{2}{(u-1)u}\,du \quad (1)$$

$$\frac{2}{(u-1)u} = \frac{A}{u-1}+\frac{B}{u} = \frac{Au+B(u-1)}{(u-1)u} \quad (\tfrac{1}{2})$$

$2 = Au + B(u-1)$ (½)

$u=0:\ 2=-B \Rightarrow B=-2$ (½)

$u=1:\ 2=A$ (½)

SANITY CHECK: $u=2:\ \frac{2}{(u-1)u}=\frac{2}{2}=1 \qquad \frac{2}{u-1}+\frac{-2}{u}=\frac{2}{1}+\frac{-2}{2}=1$ (½)

$$\int\left(\frac{2}{u-1}+\frac{-2}{u}\right)du = 2\ln|u-1| - 2\ln|u| + C = 2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C = \ln x - 2\ln(1+\sqrt{x}) + C$$

(½) (½) (1) (½) (½)

SEE ALTERNATE SOLUTION 9:30 VERSION E

Find $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx$.

SCORE: ___ / 3 POINTS

$$\frac{(-x)^5}{\sqrt{1+(-x)^4}} = \frac{-x^5}{\sqrt{1+x^4}} = -\frac{x^5}{\sqrt{1+x^4}} \quad (1)$$

Since the integrand is odd and continuous, and the interval is symmetric, $\int_{-2}^{2} \frac{x^5}{\sqrt{1+x^4}}\,dx = 0$. (1) (1)

Find $\int \frac{x^2+2}{x^3+2x^2+x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{x^2+2}{x^3+2x^2+x} = \frac{x^2+2}{x(x+1)^2} = \left[\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right] = \frac{A(x+1)^2+Bx(x+1)+Cx}{x(x+1)^2}$$ 1

$x^2+2 = A(x+1)^2+Bx(x+1)+Cx$ ½

$x=0$: $2=A$ 1

$x=-1$: $3=-C \Rightarrow C=-3$ 1

$x=1$: $3=4A+2B+C \Rightarrow B=-1$ 1

SANITY CHECK: $x=2$: $\frac{x^2+2}{x^3+2x^2+x}=\frac{6}{18}=\frac{1}{3}$ ½ $\frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2}=\frac{2}{2}+\frac{-1}{3}+\frac{-3}{9}=\frac{1}{3}$ ½

$$\int\left(\frac{2}{x}+\frac{-1}{x+1}+\frac{-3}{(x+1)^2}\right)dx = 2\ln|x|-\ln|x+1|+\frac{3}{x+1}+C$$ ½ ½ ½ ½

Find $\int \frac{x+26}{x^3+4x^2+13x}\,dx$.

SCORE: ___ / 7 POINTS

$$\frac{x+26}{x^3+4x^2+13x} = \frac{x+26}{x((x+2)^2+9)} = \left[\frac{A}{x}+\frac{B(2x+4)+C(3)}{((x+2)^2+9)}\right] = \frac{A((x+2)^2+9)+Bx(2x+4)+C(3x)}{x((x+2)^2+9)}$$ 1

$x+26 = A((x+2)^2+9)+Bx(2x+4)+C(3x)$ ½

$x=0$: $26=13A \Rightarrow A=2$ 1

$x=-2$: $24=9A-6C \Rightarrow C=-1$ 1

$x=1$: $27=18A+6B+3C \Rightarrow B=-1$ 1

SANITY CHECK: $x=2$: $\frac{x+26}{x^3+4x^2+13x}=\frac{28}{50}=\frac{14}{25}$ ½ $\frac{2}{x}+\frac{-(2x+4)-(3)}{((x+2)^2+9)}=\frac{2}{2}+\frac{-11}{25}=\frac{14}{25}$ ½

$$\int\left(\frac{2}{x}+\frac{-(2x+4)-(3)}{((x+2)^2+9)}\right)dx = 2\ln|x|-\ln(x^2+4x+13)-\tan^{-1}\frac{x+2}{3}+C$$ ½ ½ ½ ½

Find $\int (\arcsin x)^2\,dx$.

SCORE: ___ / 6 POINTS

u	dv
$(\arcsin x)^2$	1
$\frac{2\arcsin x}{\sqrt{1-x^2}}$	x
$\arcsin x$	$\frac{2x}{\sqrt{1-x^2}}$
$\frac{1}{\sqrt{1-x^2}}$	$-2\sqrt{1-x^2}$
1	-2
0	$-2x$

$$\int (\arcsin x)^2\,dx = x(\arcsin x)^2+2\sqrt{1-x^2}\arcsin x-2x+C$$ 2 2 1½ ½

SEE ALTERNATE SOLUTION 9:30 VERSION E