Section 2.3 #2 Solutions

These are the properly written solutions.
Most students can do the arithmetic, but many are sloppy or lacking in the writing.
Unfortunately, that means they won't receive full credit.
It also means that they might find the multiple choice questions on the final exam more confusing than they actually are.

To help you out, I have highlighted the steps that students often don't write that cost them points.
In order to receive full credit when the test question says "show the use of limit laws", you must include the steps pointed out in red.
(If the question does not say "show the use of limits laws", you may skip the highlighted steps.)

[a] $\lim_{x\to 2}[f(x)+g(x)]$

$= \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$ ⬅ **This step is mandatory to receive full credit**

$= 2 + 0$

$= 2$

[b] $\lim_{x\to 1^-}[f(x)+g(x)]$ ⬅ **This step is mandatory to receive full credit**

$= \lim_{x\to 1^-} f(x) + \lim_{x\to 1^-} g(x)$ ⬅ **This step is mandatory to receive full credit**

$= 1 + 2$

$= 3$

$\lim_{x\to 1^+}[f(x)+g(x)]$ ⬅ **This step is mandatory to receive full credit**

$= \lim_{x\to 1^+} f(x) + \lim_{x\to 1^+} g(x)$ ⬅ **This step is mandatory to receive full credit**

$= 1 + 1$

$= 2$

Since $\lim_{x\to 1^-}[f(x)+g(x)] \neq \lim_{x\to 1^+}[f(x)+g(x)]$,

therefore $\lim_{x\to 1}[f(x)+g(x)]$ does not exist

[c] $\lim_{x\to 0}[f(x)g(x)]$

$= \lim_{x\to 0} f(x) \times \lim_{x\to 0} g(x)$ ⬅ **This step is mandatory to receive full credit**

$= 0 \times 1.3$

$= 0$

[d] As $x \to -1^-$, ⬅ **These two lines could be replaced**

$f(x) \to -1$ and $g(x) \to 0^-$, ⬅ **with the notation** $\frac{-1}{0^-}$**. Do NOT write** $\lim_{x\to -1^-}\frac{f(x)}{g(x)} = \frac{-1}{0^-} = \infty$

so $\lim_{x\to -1^-}\frac{f(x)}{g(x)} = \infty$

As $x \to -1^+$, ⬅ **These two lines could be replaced**

$f(x) \to -1$ and $g(x) \to 0^+$, ⬅ **with the notation** $\frac{-1}{0^+}$**. Do NOT write** $\lim_{x\to -1^+}\frac{f(x)}{g(x)} = \frac{-1}{0^+} = -\infty$

so $\lim_{x\to -1^-}\frac{f(x)}{g(x)} = -\infty$

Since $\lim_{x \to -1^-} \frac{f(x)}{g(x)}$ and $\lim_{x \to -1^+} \frac{f(x)}{g(x)}$ are not equivalent,

therefore $\lim_{x \to -1} \frac{f(x)}{g(x)}$ does not exist

[e] $\lim_{x \to 2} [x^3 f(x)]$

$= \lim_{x \to 2} x^3 \times \lim_{x \to 2} f(x)$ ⬅ **This step is mandatory to receive full credit**

$= 2^3 \times 2$

$= 16$

[f] $\lim_{x \to 1} \sqrt{3 + f(x)}$

$= \sqrt{\lim_{x \to 1} [3 + f(x)]}$ ⬅ **This step is mandatory to receive full credit**

$= \sqrt{\lim_{x \to 1} 3 + \lim_{x \to 1} f(x)}$ ⬅ **This step is mandatory to receive full credit**

$= \sqrt{3 + 1}$

$= 2$