Math 1A (10:30am – 11:20am)
Midterm 1 Version C
Wed Jan 26, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS OR DIFFERENTIATION SHORTCUTS (CH 3) ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS UNLESS SPECIFICALLY ASKED FOR

The volume of water in a reservoir t hours after noon is $V(t) = \dfrac{6+t}{2+\sqrt{t}}$ million gallons. **SCORE: ___ / 25 POINTS**

[a] What was the average rate of change of the volume from noon to 4 pm ?
<u>Specify the units of your answer.</u>

$$\frac{V(4)-V(0)}{4-0} = \frac{\frac{5}{2}-3}{4} = -\frac{1}{8} \text{ MILLION GALLONS PER HOUR}$$

[b] What was the instantaneous rate of change of the volume at 9 pm ?
<u>Specify the units of your answer, and specify if the population was increasing or decreasing.</u>

$$V'(9) = \lim_{t \to 9} \frac{V(t)-V(9)}{t-9}$$

$$= \lim_{t \to 9} \frac{\frac{6+t}{2+\sqrt{t}} - 3}{t-9} \cdot \frac{2+\sqrt{t}}{2+\sqrt{t}}$$

$$= \lim_{t \to 9} \frac{t-3\sqrt{t}}{(t-9)(2+\sqrt{t})} \cdot \frac{t+3\sqrt{t}}{t+3\sqrt{t}}$$

$$= \lim_{t \to 9} \frac{\cancel{t^2-9t}\ t}{\cancel{(t-9)}(2+\sqrt{t})(t+3\sqrt{t})}$$

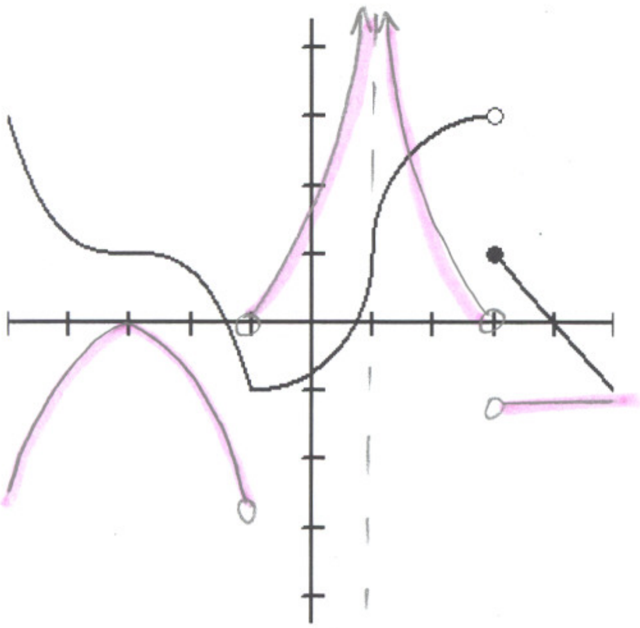
$$= \frac{1}{10} \text{ MILLION GALLONS PER HOUR (INCREASING)}$$

☆ SEE ALSO 10:30 VERSION D (NOT PREFERRED)

State the Intermediate Value Theorem. **SCORE: ___ / 5 POINTS**

SEE 7:30 VERSION A

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ **on the same axes**. SCORE: ___ / 15 POINTS



Find the **equation(s)** of the horizontal asymptote(s) of $f(x) = \dfrac{\tan^{-1} x}{e^x + 4}$. SCORE: ___ / 15 POINTS

Show the proper use of the limit laws.

$$\lim_{x \to -\infty} \frac{\tan^{-1} x}{e^x + 4}$$

$$= \frac{\lim_{x \to -\infty} \tan^{-1} x}{\lim_{x \to -\infty} (e^x + 4)}$$

$$= \frac{\lim_{x \to -\infty} \tan^{-1} x}{\lim_{x \to -\infty} e^x + \lim_{x \to -\infty} 4}$$

$$= \frac{-\frac{\pi}{2}}{0 + 4} = -\frac{\pi}{8}$$

$$\lim_{x \to \infty} \frac{\tan^{-1} x}{e^x + 4} = 0$$

$$\frac{\frac{\pi}{2}}{\infty + 4} \to \frac{\frac{\pi}{2}}{\infty}$$

$y = 0, -\frac{\pi}{8}$

State the complete definition of "the derivative (function)". SCORE: ___ / 5 POINTS

SEE 7:30 VERSION A

State the complete definition of "horizontal asymptote". SCORE: ___ / 5 POINTS

SEE 7:30 VERSION A

The number of freshmen who apply to major in a certain field depends on the average starting salary in that field. **SCORE: ___ / 15 POINTS**
Let $f = a(s)$, where f is the number of freshmen who apply, and s is the average starting salary (in thousands of dollars).

[a] What are the units of $a'(s)$?

FRESHMEN PER THOUSANDS OF DOLLARS

[b] Give the practical meaning (including units) of $a'(100) = 50$.

IF THE AVERAGE STARTING SALARY IS \$100,000, 50 MORE FRESHMEN WILL APPLY TO MAJOR FOR EACH \$1,000 INCREASE IN THE STARTING SALARY

[c] Is there a value of s_0 for which you would expect $a'(s_0) < 0$? Why or why not ?

NO. IF THE STARTING SALARY INCREASES, MORE STUDENTS WILL WANT TO MAJOR IN THE FIELD.

Consider the function $f(x) = \tan x$ on the interval $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$ with $d = 0$. **SCORE: ___ / 15 POINTS**

[a] Does this situation satisfy the conditions of the Intermediate Value Theorem ? Why or why not ?

NO. f IS NOT CONTINUOUS AT $x = \frac{\pi}{2}, \frac{3\pi}{2} \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

[b] Does this situation satisfy the conclusion of the Intermediate Value Theorem ? Why or why not ?

YES. $\tan x = 0$ AT $x = \pi \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

[c] Can the Intermediate Value Theorem be used to prove that $\tan x = 0$ somewhere in the interval $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$? Why or why not ?

NO. SINCE f IS NOT CONTINUOUS ON $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$,

THE IVT DOESN'T APPLY.

Let $f(x) = \frac{x^2 - x - 12}{9 - x^2}$.

SCORE: ___ / 25 POINTS

[a] Find all discontinuities of f.

$9 - x^2 = 0$

$x = \pm 3$

[b] Find the limit of f at each discontinuity.

Each limit should be a number, ∞ or $-\infty$. **<u>Write DNE only if the other possibilities do not apply.</u>**

$\lim_{x \to 3^+} \frac{x^2 - x - 12}{9 - x^2} = \infty$

$\frac{-6}{0^-}$

$\lim_{x \to 3^-} \frac{x^2 - x - 12}{9 - x^2} = -\infty$

$\frac{-6}{0^+}$

$\lim_{x \to 3} \frac{x^2 - x - 12}{9 - x^2}$ DNE

$\lim_{x \to -3} \frac{x^2 - x - 12}{9 - x^2}$

$= \lim_{x \to -3} \frac{(x-4)\cancel{(x+3)}}{(3-x)\cancel{(3+x)}}$

$= \frac{-7}{6}$

[c] State the type of each discontinuity of f.

X = 3 IS AN INFINITE DISCONTINUITY

X = −3 IS A REMOVABLE DISCONTINUITY

Let $f(x) = 2x^2 - 4x^3$.

SCORE: ___ / 25 POINTS

[a] Find $f'(x)$.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 4(x+h)^3 - (2x^2 - 4x^3)}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{4x^3} - 12x^2h - 12xh^2 - 4h^3 - \cancel{2x^2} + \cancel{4x^3}}{h}$

$= \lim_{h \to 0} (4x + 2h - 12x^2 - 12xh - 4h^2)$

$= 4x - 12x^2$

[b] Find the equation of the tangent line to $y = f(x)$ at $x = -1$.

$f'(-1) = -16$

$y - 6 = -16(x + 1)$