Math 1A (7:30am – 8:20am)
Midterm 1 Version A
Wed Jan 26, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS OR DIFFERENTIATION SHORTCUTS (CH 3) ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS UNLESS SPECIFICALLY ASKED FOR

The population of a city t years after the year 2000 is $p(t) = \dfrac{3+\sqrt{t}}{6+t}$ million people. **SCORE: ___ / 25 POINTS**

[a] What was the average rate of change of the population from the year 2000 to the year 2009 ?
Specify the units of your answer.

$$\frac{p(9)-p(0)}{9-0} = \frac{\frac{2}{5}-\frac{1}{2}}{9} = -\frac{1}{90} \text{ MILLION PEOPLE PER YEAR}$$

[b] What was the instantaneous rate of change of the population in 2004 ?
Specify the units of your answer, and specify if the population was increasing or decreasing.

$$p'(4) = \lim_{t\to 4} \frac{p(t)-p(4)}{t-4}$$

$$= \lim_{t\to 4} \frac{\frac{3+\sqrt{t}}{6+t}-\frac{1}{2}}{t-4} \cdot \frac{2(6+h)}{2(6+h)}$$

★ SEE ALSO 7:30 VERSION B (NOT PREFERRED)

$$= \lim_{t\to 4} \frac{2\sqrt{t}-t}{2(t-4)(6+t)} \cdot \frac{2\sqrt{t}+t}{2\sqrt{t}+t}$$

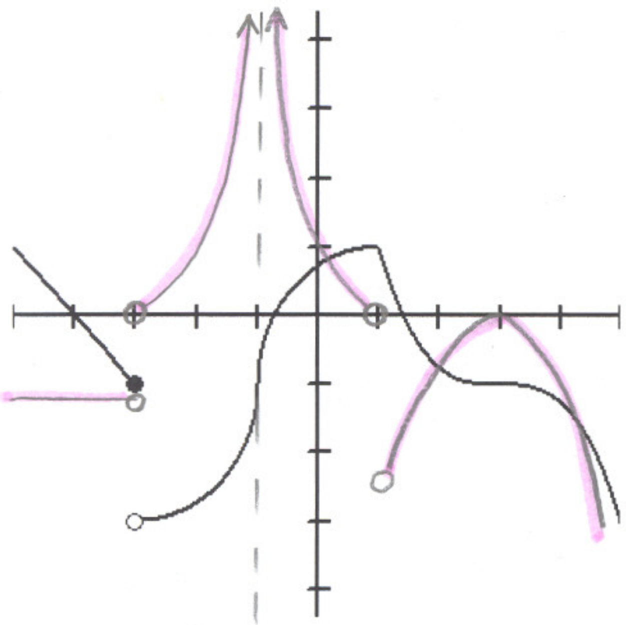
$$= \lim_{t\to 4} \frac{\cancel{4t-t^2}\ -t}{2\cancel{(t-4)}(6+t)(2\sqrt{t}+t)}$$

$$= -\frac{1}{40} \text{ MILLION PEOPLE PER YEAR (DECREASING)}$$

State the complete definition of "the derivative (function)". **SCORE: ___ / 5 POINTS**

THE DERIVATIVE OF f IS $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

The graph of $f(x)$ is shown below. Sketch a graph of $f'(x)$ **on the same axes**. **SCORE: ___ / 15 POINTS**



Find the **equation(s)** of the horizontal asymptote(s) of $f(x) = \dfrac{\tan^{-1} x}{2 + e^{-x}}$. **SCORE: ___ / 15 POINTS**

Show the proper use of the limit laws.

$$\lim_{x\to\infty} \frac{\tan^{-1}x}{2+e^{-x}} = \frac{\lim\limits_{x\to\infty} \tan^{-1}x}{\lim\limits_{x\to\infty}(2+e^{-x})} = \frac{\lim\limits_{x\to\infty}\tan^{-1}x}{\lim\limits_{x\to\infty} 2 + \lim\limits_{x\to\infty} e^{-x}} = \frac{\frac{\pi}{2}}{2+0} = \frac{\pi}{4}$$

$$\lim_{x\to-\infty} \frac{\tan^{-1}x}{2+e^{-x}} = 0$$

$$\frac{-\frac{\pi}{2}}{2+\infty} \to \frac{-\frac{\pi}{2}}{\infty}$$

$y = 0$, $y = \frac{\pi}{4}$

State the complete definition of "horizontal asymptote". **SCORE: ___ / 5 POINTS**

f HAS A HORIZONTAL ASYMPTOTE AT $y=b$ IF $\lim\limits_{x\to\infty} f(x) = b$ OR $\lim\limits_{x\to-\infty} f(x) = b$

State the Intermediate Value Theorem. **SCORE: ___ / 5 POINTS**

IF f IS CONTINUOUS ON $[a,b]$
AND d IS BETWEEN $f(a)$ AND $f(b)$
THEN THERE IS A $c \in (a,b)$ SUCH THAT $f(c) = d$

The number of students who register for classes at a community college depends on the cost per unit for classes. **SCORE: ___ / 15 POINTS**
Let $s = p(c)$, where s is the number of students (in hundreds of students), and c is the cost per unit (in dollars).

[a] What are the units of $p'(c)$?

HUNDREDS OF STUDENTS PER DOLLAR

[b] Give the practical meaning (including units) of $p'(18) = -11$.

IF CLASSES COST \$18 PER UNIT,
1100 FEWER STUDENTS WILL REGISTER
FOR EACH \$1 PER UNIT INCREASE IN THE COST OF CLASSES

[c] Is there a value of c_0 for which you would expect $p'(c_0) > 0$? Why or why not ?

NO. IF THE COST PER UNIT INCREASES, FEWER STUDENTS WILL BE ABLE TO AFFORD TO PAY, SO THE NUMBER OF STUDENTS WILL DECREASE.

Consider the function $f(x) = \dfrac{7}{x^2 - 2}$ on the interval $[-3, 1]$ with $d = -4$. **SCORE: ___ / 15 POINTS**

[a] Does this situation satisfy the conditions of the Intermediate Value Theorem ? Why or why not ?

NO. f IS NOT CONTINUOUS AT $x = -\sqrt{2} \in [-3, 1]$

[b] Does this situation satisfy the conclusion of the Intermediate Value Theorem ? Why or why not ?

YES. $\dfrac{7}{x^2-2} = -4$

$7 = -4x^2 + 8$

$4x^2 = 1$

$x = \pm\frac{1}{2} \in [-3, 1]$

[c] Can the Intermediate Value Theorem be used to prove that $\dfrac{7}{x^2 - 2} = -4$ somewhere in the interval $[-3, 1]$? Why or why not ?

NO. SINCE f IS NOT CONTINUOUS ON $[-3, 1]$,
THE IVT DOESN'T APPLY

Let $f(x) = \dfrac{x^2 - x - 6}{4 - x^2}$.

SCORE: ___ / 25 POINTS

[a] Find all discontinuities of f.

$4 - x^2 = 0$

$x = \pm 2$

[b] Find the limit of f at each discontinuity.
Each limit should be a number, ∞ or $-\infty$. **Write DNE only if the other possibilities do not apply.**

$\lim\limits_{x \to 2^+} \dfrac{x^2-x-6}{4-x^2} = \infty$

$\dfrac{-6}{0^-}$

$\lim\limits_{x \to 2^-} \dfrac{x^2-x-6}{4-x^2} = -\infty$

$\dfrac{-6}{0^+}$

$\lim\limits_{x \to 2} \dfrac{x^2-x-6}{4-x^2}$ DNE

$\lim\limits_{x \to -2} \dfrac{x^2-x-6}{4-x^2}$

$= \lim\limits_{x \to -2} \dfrac{(x-3)\cancel{(x+2)}}{(2-x)\cancel{(2+x)}}$

$= \dfrac{-5}{4}$

[c] State the type of each discontinuity of f.

X=2 IS AN INFINITE DISCONTINUITY

X=-2 IS A REMOVABLE DISCONTINUITY

Let $f(x) = 4x^2 - 2x^3$.

SCORE: ___ / 25 POINTS

[a] Find $f'(x)$.

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{4(x+h)^2 - 2(x+h)^3 - (4x^2 - 2x^3)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{2x^3} - 6x^2h - 6xh^2 - 2h^3 - \cancel{4x^2} + \cancel{2x^3}}{h}$

$= \lim\limits_{h \to 0} (8x + 4h - 6x^2 - 6xh - 2h^2)$

$= 8x - 6x^2$

[b] Find the equation of the tangent line to $y = f(x)$ at $x = -1$.

$f'(-1) = -14$

$y - 6 = -14(x+1)$